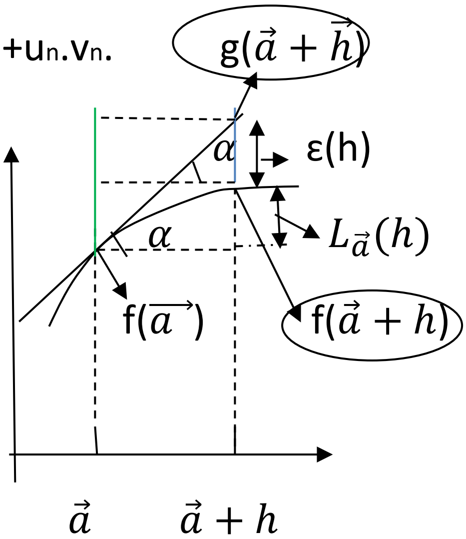
DERIVADES PARCIALS EN 3-D:

Sabent que $<\vec{u}|\vec{v}>= u_1.v_1 + u_2.v_2+...+u_n.v_n$.



I com que segons Cauchy-Schwarz

$|\vec{x}.\vec{y}| = \langle\vec{x}|\vec{y}\rangle \leq \|\vec{x}\|.\|\vec{y}\|,$

$<D_n f\left(\vec{a}\right)|\ \vec{h}> \leq \left\|D_n f\left(\vec{a}\right)\right\|.\|\vec{h}\|$

On n=2

$\langle\vec{x}|\vec{y}\rangle = |\sum_{i=1}^{n} x_i.y_i|$

Si f és diferenciable, $\vec{h} = h.\hat{u}$ i $L_{\vec{a}}(\hat{u})$ és el mateix que $Df_{\vec{a}}(\hat{u})$

i $L_{\vec{a}}(h) = <D_n f\left(\vec{a}\right)|\ \vec{h}>$ i $D_n f\left(\vec{a}\right)$ són les pendents. definim

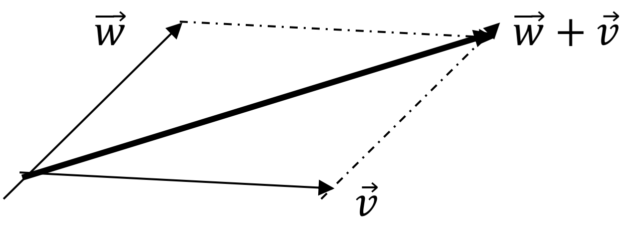
$\vec{\nabla}f(\vec{a}) = (D_1 f\left(\vec{a}\right), D_2 f\left(\vec{a}\right))$ i $\vec{h} = (h_1, h_2)$

quan tractem $\hat{u} = \hat{e}_n$

$L_{\vec{a}}(\vec{h}) = L_{\vec{a}}(\sum_{n=1}^{2} h_n\hat{e}_n) = \sum_{n=1}^{2} h_n.L_{\vec{a}}(\hat{e}_n)$

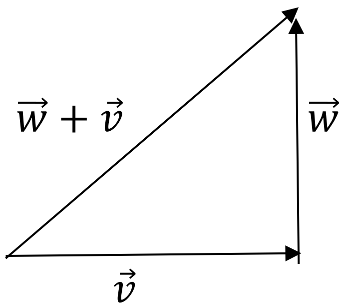
$Df_{\vec{a}}(\hat{e}_n) = L_{\vec{a}}(\hat{e}_n) = D_n f\left(\vec{a}\right)$

sabem també que la suma de vectors dóna un vector, i és:

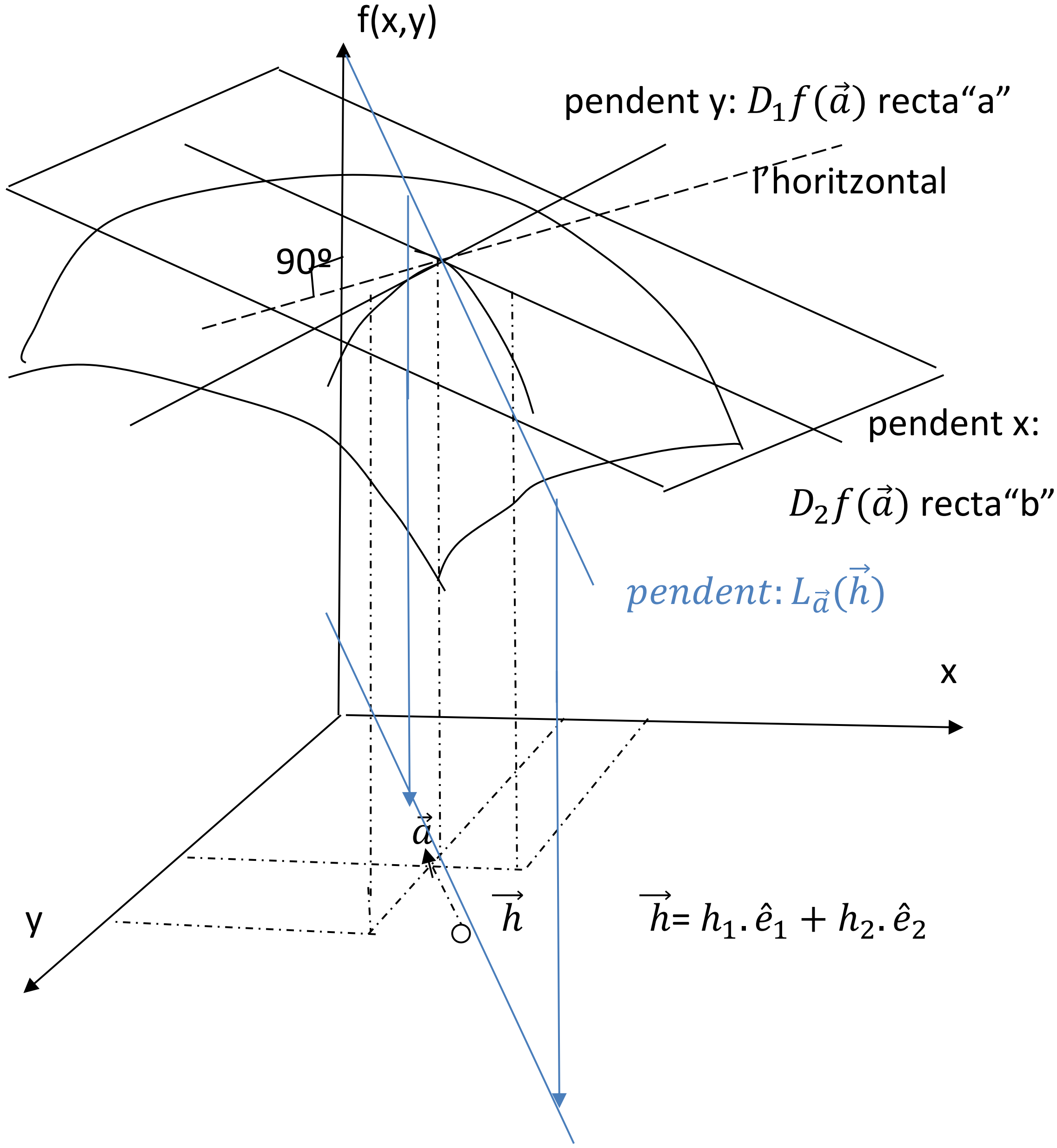


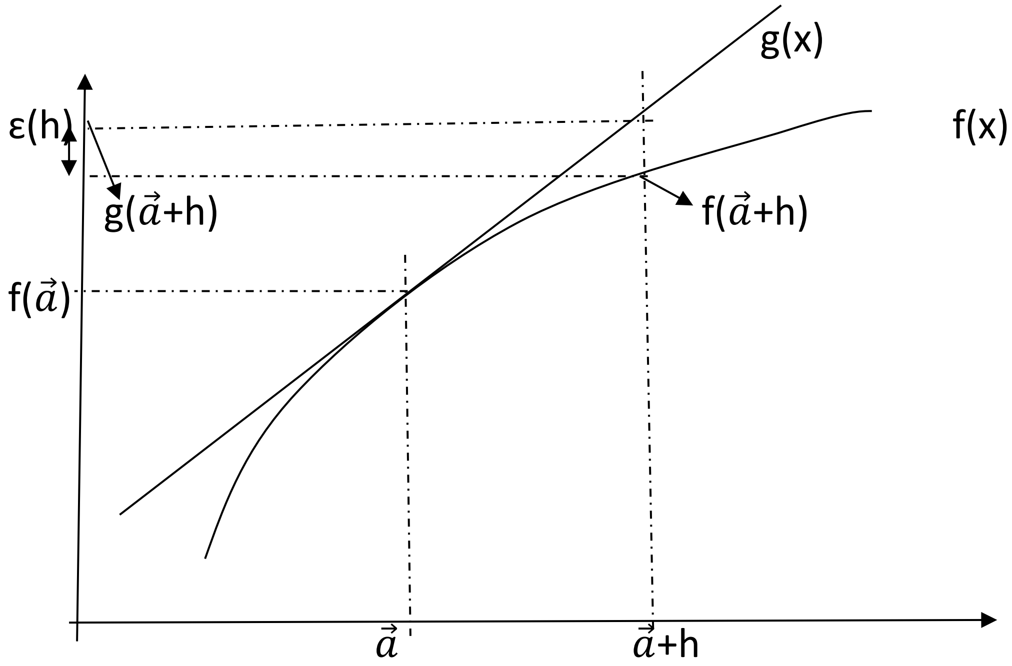
(ȝ)

mentre que si treballem amb distàncies (i estem davant de triangles rectangles):



$$\|\vec{w} + \vec{v}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2$$


f(x,y)
pendent y: $D_1 f(\vec{a})$ recta“a”
l’horitzontal
90º
pendent x:
$D_2 f(\vec{a})$ recta“b”
$pendent: L_{\vec{a}}(\vec{h})$
x
y
$\vec{a}$
$\vec{h}$
$\vec{h} = h_1.\hat{e}_1 + h_2.\hat{e}_2$



g($\vec{a}$+h)- f($\vec{a}$)= f'($\vec{a}$).h   ε(h)= g($\vec{a}$+h)- f($\vec{a}$+h)

$$\lim_{h\to 0}\left|\frac{\varepsilon(\mathrm{h})}{h}\right| = 0 = \lim_{h\to 0}\left|\frac{\mathrm{g}(\vec{a}+\mathrm{h}) - \mathrm{f}(\vec{a}+\mathrm{h})}{h}\right| = \lim_{h\to 0}\frac{f(\vec{a})+\mathrm{f}'(\vec{a}).\mathrm{h}-\mathrm{f}(\vec{a}+\mathrm{h})}{h} = 0$$

amb els signes canviats: - f'$(\vec{a})$= $\lim_{h\to 0}\frac{f(\vec{a})-\mathrm{f}(\vec{a}+\mathrm{h})}{h}$

$L_{\vec{a}}\big(\vec{h}\big) = L_{\vec{a}}(h.\hat{u}) = h.L_{\vec{a}}(\hat{u})$ quan $\lim_{\vec{h}\to 0}\frac{\left|f(\vec{a}+\vec{h})-f(\vec{a})-L_{\vec{a}}(\vec{h})\right|}{\|\vec{h}\|}$

i, com hem vist, $\vec{h}$= $h.\hat{u}$

$L_{\vec{a}}(\vec{h})$= $L_{\vec{a}}(\sum_{n=1}^{2} h_n\hat{e}_n) = \sum_{n=1}^{2} h_n.L_{\vec{a}}(\hat{e}_n)$

$Df_{\vec{a}}(\hat{e}_n) = L_{\vec{a}}(\hat{e}_n) = D_nf\left(\underset{a}{\to}\right)$=pendent, i la suma de les dues rectes prependiculars “a” i “b” dóna una resultant, segons (ƺ).