

APROXIMACIÓ D'STIRLING.

$$N! \sim N \cdot \ln N - N$$

$$P(x) = e^x = 1 + x \cdot f'(a) + \frac{x^2 \cdot f''(a)}{2!} + \frac{x^3 \cdot f'''(a)}{3!} + \frac{x^4 \cdot f^{IV}(a)}{4!} + \dots =$$

$$= \sum_{n=0}^{\infty} (+1)^n \cdot \frac{x^n}{n!} \cdot f^n(a)$$

$$P(x) = e^{-x} = 1 - x \cdot f'(a) + \frac{x^2 \cdot f''(a)}{2!} - \frac{x^3 \cdot f'''(a)}{3!} + \frac{x^4 \cdot f^{IV}(a)}{4!} + \dots =$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^n}{n!} \cdot f^n(a)$$

$$\text{D'altra banda, } P(x) = \sum_{n=0}^{\infty} a_n \cdot x^n$$

Origen de la transformada de Laplace:

$$\sum_{n=0}^{\infty} a_n \cdot x^n = \Delta x \text{ (convergent)}$$

$$\sum_{n=0}^{\infty} f(n) \cdot x^n = \Delta x$$

$f(N) \rightarrow R$ on $N = n^{\text{o}}\text{'s naturals}$ i $R = n^{\text{o}}\text{'s reals}$

$$n \rightarrow f(n) = a_n$$

en un espai continu: suposem t contínua i $0 < t < \infty$

$$\int_0^{\infty} f(t) \cdot x^{-t} \cdot dt = F(x) \text{ quan } x = e^{\ln x} \text{ i } x^{-t} = [e^{\ln x}]^{-t}$$

llavors la solució de $F(x)$ ha de ser de naturalesa real i $f(x)$ no ha de créixer més que x^{-t} . funció gamma: $\Gamma(x)$.

$$0 < x < 1 \rightarrow \ln x < 0$$

com que treballem amb termes positius, definim $-\xi = \ln x$

o sigui que $\int_0^{\infty} f(t) \cdot x^{-\xi t} \cdot dt = \mathcal{L}(f(t))$ que és la transformada de Laplace.

sabent que $\int_a^b f + g = \int_a^b f + \int_a^b g$

perquè la sèrie convergeixi, $f(t)$ no ha de créixer més que $t^n = t^{x-1}$

$$\text{i } a_n = \sum_{n=0}^{\infty} f(t) \cdot t^n \cdot dt$$

$$\text{i } \ln x = y \quad e^y = x \quad e^{\ln x} = x \quad \ln(x^p) = p \cdot \ln x$$

$$\text{SEGONS EULER: } \sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\text{i } 1 = \frac{\int_{n=0}^{\infty} e^{-t} \cdot t^n}{n!} \cdot dt$$

MENTRE QUE SEGONS BOREL:

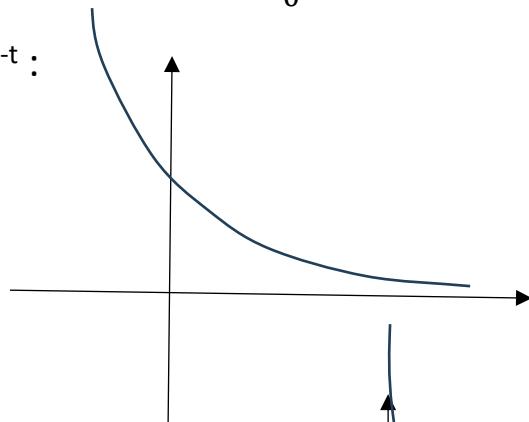
$$\int_0^{\infty} dt \cdot e^{-t} \cdot \left(\sum_{n=0}^{\infty} (+1)^n \cdot \frac{t^n}{n!} \cdot C \right) \quad \text{on } C \equiv f^n(a)$$

A més: $\int_0^{\infty} dt \cdot e^{-t} \cdot t^n$ on $n = x-1$ només convergeix quan $x > 1$

$$\int_a^b f(t) \cdot dt = \sum_{n=0}^{\infty} a_n \int_a^b t^{x-1} \cdot dt$$

$$\int_0^{\infty} dt \cdot e^{-t} \cdot t^{x-1} \equiv \int_0^{\infty} dt \cdot e^{-t} \cdot \left(\sum_{n=0}^{\infty} (-1)^n \cdot \frac{t^n}{n!} \cdot C \right)$$

$y = e^{-t}$:



$y = f(t) = x^{t-1}$ on $t < 1$



$$\int_0^\infty x^n \cdot e^{-x} \cdot dx =$$

$$x^n = t \rightarrow \ln(x^n) = Lnt \rightarrow x = (e^{Lnt})^{\frac{1}{n}} \rightarrow$$

$$x = e^{(Lnt)/n} \rightarrow dx = e^{(Lnt)/n} \cdot \frac{1}{t^{1/n}} \cdot \frac{t^{(\frac{1}{n}-1)}}{n} \cdot dt$$

$$\int_0^\infty e^{Lnt} \cdot e^{-t^{1/n}} \cdot e^{Lnt^{1/n}} \cdot \frac{1}{n \cdot t} dt = \int_0^\infty e^{Lnt + Lnt^{1/n} - t^{1/n}} \cdot \frac{1}{n \cdot t} dt =$$

$$= \int_0^\infty e^{Lnt + Lnt^{1/n} - t^{1/n}} \cdot \frac{1}{n \cdot t} dt = \int_0^\infty e^{Ln(t \cdot t^{\frac{1}{n}}) - t^{1/n}} \cdot \frac{1}{n \cdot t} dt =$$

$$t^{1/n} = u \quad t = u^n \quad dt = n \cdot u^{n-1} \cdot du$$

$$= \int_0^\infty e^{u \cdot Ln(u^n) - u} \cdot \frac{1}{n \cdot u^n} \cdot n \cdot u^{n-1} \cdot du = \int_0^\infty e^{u \cdot (nLnu - 1)} \cdot \frac{1}{n \cdot u} \cdot du =$$

quan $n = 1$, aprox.

Stirling

$$= \int_0^\infty \frac{e^{u(nLnu - 1)}}{u} du$$

Que com hem dit abans només és operativa quan $0 < x < 1$

$$0 < u < 1$$

$$\int_0^1 \frac{e^{u!}}{u} du \quad ja que x, t, u són variables directament proporcionals.$$

Atenció: $I = \int_a^b \frac{e^{x!}}{x} dx$ substitució $e^{x!} = u$,
 $du = e^{x!} \cdot \int_0^\infty t^x \cdot lnt \cdot e^{-t} dt \cdot dx$, $\frac{1}{x} dx = dv$ $v = \ln x$

Ja que:

$$\begin{aligned}\frac{d}{dx}(x!) &= \frac{d}{dx}(\Gamma(x+1)) = \\ &= \frac{d}{dx}\left(\int_0^\infty t^{x+1-1} \cdot e^{-t} dt\right) = \int_0^\infty \frac{d}{dx} t^{x+1-1} \cdot e^{-t} dt = \int_0^\infty t^x \cdot lnt \cdot e^{-t} dt\end{aligned}$$

$$i \quad I = e^{x!} \cdot \ln x - \int_a^b \ln x \cdot e^{x!} \int_0^\infty t^x \cdot lnt \cdot e^{-t} dt \cdot dx$$

$$i \text{ ara } \int_0^\infty t^x \cdot lnt \cdot e^{-t} dt \equiv I'(c) = \int_0^\infty t^c \cdot lnt \cdot e^{-t} dt,$$

$$\text{ja que } \frac{d}{dc}(t^c) = t^c \cdot lnt \quad i \quad I(c) = \int_0^\infty t^c \cdot e^{-t} dt = \Gamma(c+1)$$

$$\Gamma'(c+1) = \int_0^\infty \frac{d}{dc}(t^c) \cdot e^{-t} dt \quad c=x$$

$$I = e^{x!} \cdot \ln x - \int_a^b \ln x \cdot e^{x!} \cdot \Gamma'(x+1) \cdot dx, \quad \Gamma'(x+1) = (x \cdot \Gamma(x))'$$

$$\frac{d}{dx}(\Gamma(x+1)) = \Gamma(x) + x\Gamma'(x), \quad \Gamma(x+1) = x! \quad \frac{d}{dx}(x!)$$

$$I = e^{x!} \cdot \ln x - \int_a^b \ln x \cdot e^{x!} (\Gamma(x) + x\Gamma'(x)) dx$$

On $c=m-1$

$$\begin{aligned}\int_a^b \ln x \cdot e^{x!} (\Gamma(c) + x\Gamma'(c)) dx &= \int_a^b \ln x \cdot e^{x!} \Gamma(m) dx - \\ &- \int_a^b \ln x \cdot e^{x!} \cdot c \cdot \int_0^\infty t^{m-1} \cdot lnt \cdot e^{-t} dt \cdot dx = \\ &= \int_a^b \ln x \cdot e^{x!} \Gamma(m) dx - \int_a^b \ln x \cdot e^{x!} \cdot c \cdot \Gamma'(c) dx \\ \text{on } \Gamma'(c) &= \ln t \cdot \Gamma(c) \quad i \quad \Gamma(m) = (m-1)!\end{aligned}$$

$$\text{i ara, } I = e^{x!} \cdot \ln x \Big|_a^b - \int_a^b \ln x \cdot e^{x!} (m-1)! dx -$$

$$- \int_a^b \ln x \cdot e^{x!} \cdot (m-1) \cdot \ln t \cdot (m-1)! dx.$$

$$\text{i ara apliquem de nou la substitució: } \ln x = u \quad du = \frac{1}{x} dx$$

$$\text{i } dv = e^{x!} (m-1)! dx = e^{x!} (\Gamma(m)/\ln t) dx, \quad v = e^{x!}/\ln t$$

$$\begin{aligned} \int_a^b \ln x \cdot e^{x!} (m-1)! dx &= \ln x \cdot e^{x!} / \ln t \Big|_a^b - \int_a^b (e^{x!} / \ln t) \cdot (1/x) dx = \\ &= \ln x \cdot e^{x!} / \ln t \Big|_a^b - I / \ln t. \end{aligned}$$

$$\text{mentre que: } \int_a^b \ln x \cdot e^{x!} \cdot (m-1) \cdot \ln t \cdot (m-1)! dx =$$

$$= \ln t \cdot (m-1) \cdot \left[\ln x \cdot \frac{e^{x!}}{\ln t} \Big|_a^b - I / \ln t \right]$$

Per tant, concloem que:

$$I = e^{x!} \cdot \ln x \Big|_a^b - \ln x \cdot \frac{e^{x!}}{\ln t} \Big|_a^b + I / \ln t - (m-1) \cdot \ln x \cdot e^{x!} \Big|_a^b + (m-1) \cdot I =$$

$$I - I / \ln t - (m-1) \cdot I = \ln x \cdot \frac{e^{x!}}{\ln t} \Big|_a^b \left[1 - \frac{1}{\ln t} - (m-1) \right]$$