APROXIMACIÓ D'STIRLING.

LnN!~ N.lnN- N

P(x)= e^{x}= 1+x.f'(a)+ $\frac{x^2.f''(a)}{2!} + \frac{x^3.f'''(a)}{3!} + \frac{x^4.f^{IV}(a)}{4!}$ +...=

= $\sum_{n=0}^{\infty}(+1)^n.\frac{x^n}{n!}.f^n(a)$

P(x)= e^{-x}= 1- x.f'(a)+ $\frac{x^2.f''(a)}{2!} - \frac{x^3.f'''(a)}{3!} + \frac{x^4.f^{IV}(a)}{4!}$ +...=

= $\sum_{n=0}^{\infty}(-1)^n.\frac{x^n}{n!}.f^n(a)$

D'altra banda, P(x)= $\sum_{n=0}^{\infty} a_n.x^n$

Origen de la transformada de Laplace:

$\sum_{n=0}^{\infty} a_n.x^n = \Delta x$ (convergent)

$$\sum_{n=0}^{\infty} f(n).x^n = \Delta x$$

f(N) → R on N= nº's naturals i R= nº's reals

n → f(n)= a_n

en un espai continu: suposem t contínua i 0<t<∞

$\int_0^{\infty} f(t).x^{-t}.dt = F(z)$ quan x= e^{lnx} i $x^{-t} = [e^{lnx}]^{-t}$

llavors la solució de F(z) ha de ser de naturalesa real.

F(z): funció gamma: Γ(z).

0<z<1 → lnz<0 ja que f(t)= t^{z-1}

com que treballem amb termes positius, definim ξ = lnx

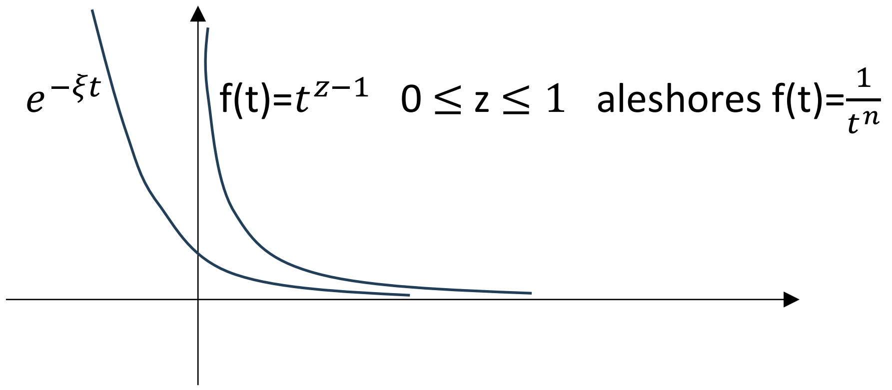
o sigui que $\int_0^\infty f(t).x^{-\xi t}.dt = \mathfrak{L}(f(t))$ que és la transformada de Laplace.

sabent que $\int_a^b f + g = \int_a^b f + \int_a^b g$

perquè $\int_0^\infty f(t).e^{-\xi t}.dt$ convergeixi, $e^{-\xi t}$ no ha de créixer més que f(t)

i $a_n = \sum_{n=0}^\infty f(t).t^n.dt$ "n=z-1"

i   lnx= y $e^y = x$ $e^{lnx} = x$ ln(x^p) =p.lnx



SEGONS EULER: $\sum_{n=0}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$

i   1= $\frac{\int_{n=0}^\infty e^{-t}.t^n}{n!}.dt$

MENTRE QUE SEGONS BOREL:

$\int_0^\infty dt.e^{-t}.(\sum_{n=0}^\infty (+1)^n.\frac{t^n}{n!}.C)$ on $C \equiv f^n(a)$

A més: $\int_0^\infty dt.e^{-t}.t^n$ on n= z-1 només convergeix quan z <1

$\int_a^b f(t).dt = \sum_{n=0}^\infty a_n \int_a^b t^{z-1}.dt$

$\int_0^\infty dt.e^{-t}.t^{z-1} \equiv \int_0^\infty dt.t^{z-1}.(\sum_{n=0}^\infty (-1)^n.\frac{t^n}{n!}.C)$

$\int_0^\infty x^n . e^{-x} . dx$=

$x^n = t \rightarrow \quad Ln(x^n) = Lnt \;\rightarrow\; x = (e^{Lnt})^{\frac{1}{n}} \rightarrow$

$x = e^{(Lnt)/n} \rightarrow$ dx= $e^{(Lnt)/n} . \frac{1}{t^{1/n}} . \frac{t^{(\frac{1}{n}-1)}}{n} . dt$

$\int_0^\infty e^{Lnt} . e^{-t^{1/n}} . e^{Lnt^{1/n}} . \frac{1}{n.t}$ dt=$\int_0^\infty e^{Lnt+Lnt^{1/n}-t^{1/n}} . \frac{1}{n.t}$ dt=

= $\int_0^\infty e^{Lnt+Lnt^{1/n}-t^{1/n}} . \frac{1}{n.t}$ dt= $\int_0^\infty e^{Ln(t.t^{\frac{1}{n}})-t^{1/n}} . \frac{1}{n.t}$ dt=

$t^{1/n} = u$ t=u^n dt=n.u^{n-1}.du t→ 0 u→0

t→ ∞ u→ ∞

= $\int_0^\infty e^{Ln(u^{n+1})-u} . \frac{1}{n.u^n}$. n.$u^{n-1}$.du= $\int_0^\infty e^{u.(Lnu-1)} . \frac{1}{u}$.du

n+1= u, dn= du, $\int_0^\infty e^{lnu!} . \left(\frac{1}{u}\right) . du$

$x^{n.x}$ dx= dv → $\int_0^1 x^{n.x} = \int_0^1 dv \;\rightarrow$ v= $\sum_{k=0}^\infty \frac{(-1)^k . n^k}{(k+1)^{k+1}}$

deducció que aquí

atenció que l'Aprox. Stirling diu: LnN!~ N.lnN- N

per tant $\int_0^\infty e^{lnu!} . \left(\frac{1}{u}\right) . du$=$\int_0^\infty u! . \left(\frac{1}{u}\right) . du$= $\int_0^\infty (u-1)! . du$

D'altra banda, atenció:

$\frac{d}{dx}(x!) = \frac{d}{dx}(\Gamma(x+1))$=

=$\frac{d}{dx}\left(\int_0^\infty t^{x+1-1} . e^{-t} dt\right)$=$\int_0^\infty \frac{d}{dx} t^{x+1-1} . e^{-t} dt$=$\int_0^\infty t^x . lnt . e^{-t} dt$

Integral de Riemann

$$\chi = \int_0^a (u-1)!.\,du = \lim_{m\to a} \sum_{u=0}^{m} (u-1)!.\Delta u$$

$$\int_0^a .\,\Gamma(u).\,du = \int_0^a \left(\int_0^\infty e^{-r} r^{u-1}.\,dr\right) du =$$

$$= \int_0^\infty e^{-r} \left(\int_0^a r^{u-1} du\right).\,dr$$

Si m→ a ∃ χ mentre que si m→ ∞ ∄ χ

Series geomètriques: $\sum_{u=0}^{m} a_u t^u$ =S_u on $a_0 = a_1 = \cdots = a_n$

S_u= $a_0 + a_1.t + a_2.t^2 + a_3.t^3 + \cdots + a_m.t^m$

-t.S_u= -$a_0.t - a_1.t^2 - a_2.t^3 - a_3.t^4 - \cdots - a_m.t^{m+1}$

S_u (1-t)= $a_0 - a_n.t^{n+1}$

a_p=$a_k.t^{p-k}$, $a_p = r.a_{p-1}$, si $a_0 = a_1 = \cdots = a_p$= 1

S_u= $\frac{a_0 - a_0.t^u}{1-t}$ = $\frac{1-1t^u}{1-t}$ =$\frac{t^u-1}{t-1}$ convergent quan $|t|$< 1

-1< t<1

Per tant: $\int_0^a r^{u-1} du = \lim_{m\to a} \sum_{u=0}^{m} r^{u-1}$.Δu

On $a_u = du = \Delta$u = ctnt. , i suposem = 1

$\sum_{u=0}^{m} a_u.r^{u-1}$= $\sum_{u=0}^{m} a_u.\frac{r^u}{r}$ cosa que usant la sèrie geomètrica s'obté S_u= $\frac{1-1r^{u+1}}{r(1-r)}$

$- r^{u+1} \qquad\qquad\qquad +1 \quad \underline{\big|\, - r^2 + r}$

$r^{u+1} - r^u \qquad\qquad\qquad\qquad\qquad r^{u-1} + r^{u-2}$

$r^u - r^{u-1} \quad +1$

...

O sigui que: $\frac{1-1r^{u+1}}{r(1-r)} = (r^{u-1} + r^{u-2}) + \frac{-r^{u-1}+1}{-r^2+r}$

L=$\int_0^\infty e^{-r}\left[(r^{u-1} + r^{u-2}) + \frac{-r^{u-1}+1}{-r^2+r}\right].dr =$

$\frac{-r^{u-1}+1}{-r^2+r} = \frac{A}{r} + \frac{B}{1-r}$ A(1-r)+Br= $-r^{u-1} + 1$

sabem també que u=n+1, per tant, si n= 1, **u= 2.**

A= 1 i r(B-A)= -r B= -1+1= 0

per tant $\int_0^\infty e^{-r}.r^{u-1}.dr + \int_0^\infty e^{-r}.r^{u-2}.dr + \int_0^\infty e^{-r}.\frac{1}{r}.dr$

$\int_0^\infty e^{-r}.r^{2-1}.dr + \int_0^\infty e^{-r}.r^{2-2}.dr + \int_0^\infty e^{-r}.\frac{1}{r}.dr$

L= $\Gamma(2) - \underbrace{\cancel{e^{-r}\Big|_0^\infty}}_{0} + \int_0^\infty e^{-r}.\frac{1}{r}.dr$

$\int e^{-r}.\frac{1}{r}.dr = \longrightarrow (1/\mathrm{r}).(-e^{-r}) - \int_0^\infty (-e^{-r}).-1/r^2 dr$

j= 1/r dj= $-1/r^2$

di= $e^{-r}dr$ i= $-e^{-r}$

$\longrightarrow \quad (1/\mathrm{r}).(-e^{-r}) - (-1/r^2).\, e^{-r} + \int_0^\infty (e^{-r}).(2/r^3)dr$

$j_2 = -1/r^2 \quad dj_2 = 2/r^3$

$di_2 = -e^{-r}dr \quad i_2 = e^{-r}$

al anar integrant per parts successivament, obtenim una sèrie següent:

$(1/r).(-e^{-r})$ - $(-1/r^2). e^{-r}$ + $(2/r^3).(- e^{-r}) + \ldots + \int_0^{\infty}$ dr

On situo "k" enlloc de "n" (per no repetir):

$\int e^{-r}.\frac{1}{r}.dr = \sum_{k=1}^{n} \frac{(-1)^k e^{-r} k!}{r^k}.1 + \int \frac{(-1)^{k+1} e^{-r}(k+1)!}{r^{k+1}}.$dr

Representat com a: $\sum_{k=1}^{n} \frac{(-1)^k e^{-r}.k!}{r^k}$+ $R_{k+1}(r)$

Recordem que vam concloure que S_u era convergent només si $|r| < 1$

Mentre que de "k+1" a "m" on $m \to \infty$

$R_{k+1}(r) = \int_{n=k+1}^{\infty} \frac{(-1)^n e^{-r}.(n)!}{r^n}$ dr= $\lim_{m\to\infty} \sum_{k+1}^{m} \frac{(-1)^{k+1} e^{-r}(k+1)!}{r^{k+1}} \to$

$\to (-1)^{\infty} e^{-r}(\infty)!. r^{-(\infty)} \to \infty.0$ suposant que tot nº multiplicat per 0 és 0.

Per tant L= $\Gamma(2) + \sum_{k=1}^{n} \frac{(-1)^k e^{-r}.k!}{r^k}$