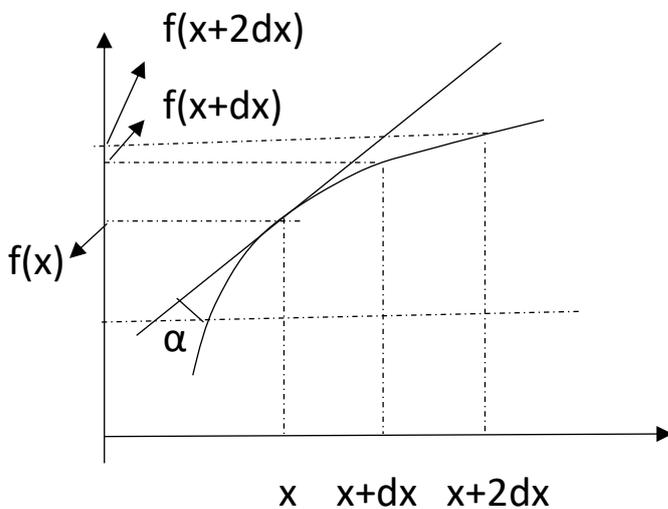


## Expressions of derivatives of degree >2:

We know that  $f'(x) = \frac{f(x+dx) - f(x)}{dx} = \tan \alpha$

and  $f(x+dx) = f(x) + f'(x).dx/1! + f''(x).dx^2/2! + f'''(x).dx^3/3! + \dots$

(when  $x=0$ )



and  $f''(x) = \frac{[f'(x+dx).dx - f'(x).dx]dx - [f(x+dx) - f(x)]dx}{dx^2}$

$f''(x) = f'(x+dx) - f'(x)$  since the derivative of  $dx$  is zero ( $dx=0$ ).

$f''(x) = \frac{f(x+2dx)dx - f(x+dx)dx}{dx} - \frac{f(x+dx)dx - f(x)dx}{dx}$

$f''(x) = f(x+2dx) - 2[f(x+dx)] + f(x)$

$f'''(x) = \frac{[f''(x+dx)dx - f''(x).dx]dx - [f'(x+dx) - f'(x)]dx}{dx^2}$

$f'''(x) = f''(x+dx) - f''(x) = f'(x+2dx) - f'(x+dx) - [f'(x+dx) - f'(x)] =$

$=$

$\frac{[f(x+3dx)dx - f(x+2dx)dx] - 2[f(x+2dx)dx - f(x+dx)dx] + [f(x+dx)dx - f(x)dx]}{dx}$

then  $f'''(x) = f(x+3dx) - 3[f(x+2dx)] + 3[f(x+dx)] - f(x)$

if we go further, we will calculate  $f^{IV}(x)$ !!

it is supposed to be  $= f(x+4dx) - 4[f(x+3dx)] + 6[f(x+2dx)] - 4[f(x+dx)] + f(x)$  (?).

verification:

$$f(x) = -2x^3 + x^2 - 4 \text{ when } x = -2 \text{ and } dx = 1$$

$$f'(x) = -6x^2 + 2x \quad f''(x) = -12x + 2 \quad f'''(x) = -12$$

$$-1 \cdot f(x) = -f(-2) = -1 \cdot (16 + 4 - 4) = -16 < 0$$

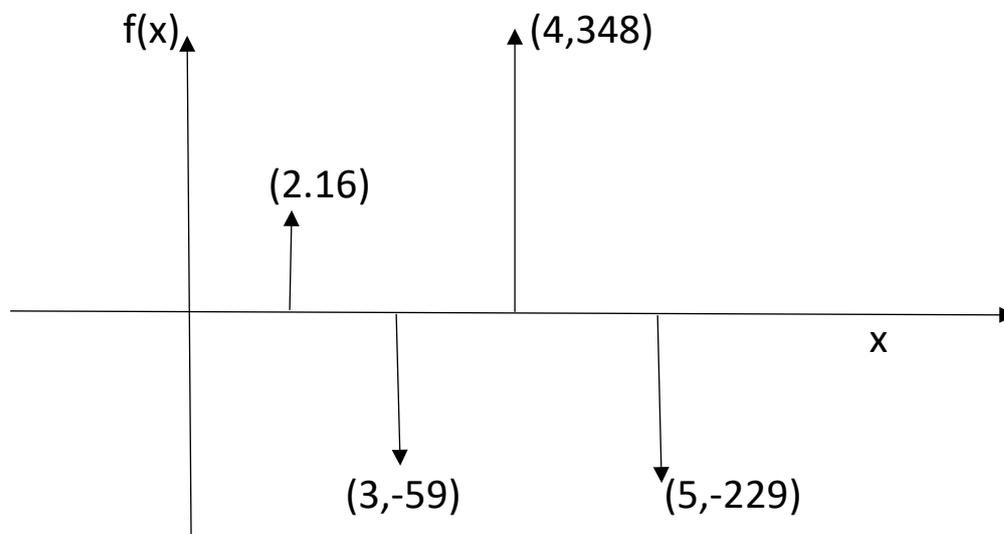
$$+3 \cdot f(x+dx) = 3 \cdot f(-1) = 3(2 + 1 - 4) = -3 < 0$$

$$-3 \cdot f(x+2dx) = -3 \cdot f(0) = +12 > 0$$

$$+1 \cdot f(x+3dx) = +1 \cdot f(1) = -5 < 0$$

$$-12 = -16 - 3 + 12 - 5 = -12$$

The sums give -12. Let's put a value of  $x$  and  $dx$  at random and see what result it gives.



3 points of intersection with the x-axis: 3 solutions to the equation.

Another example:

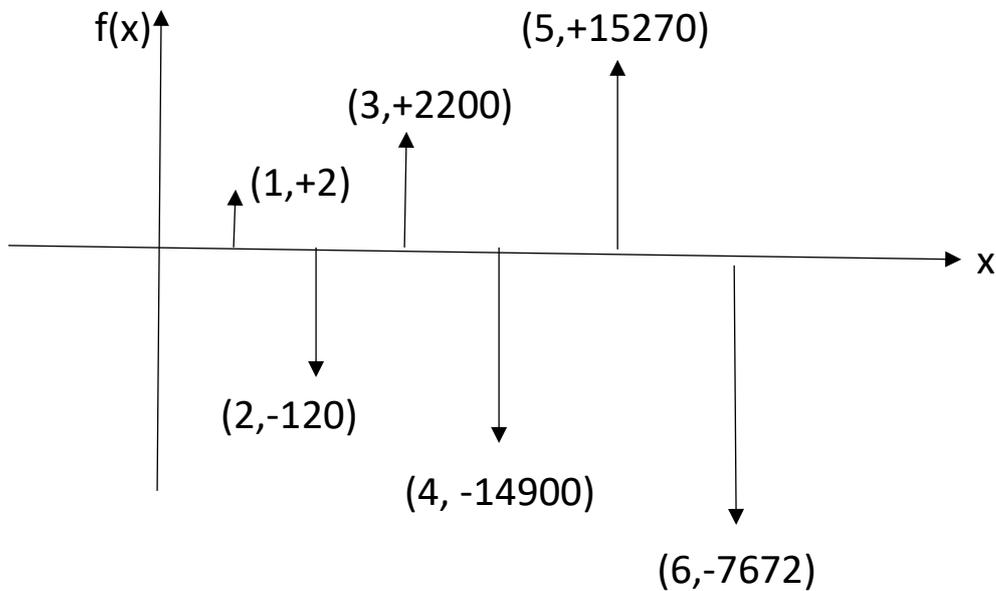
$$f(x) = x^5 - 3x^2 + 4x = 1, dx = 1$$

$$f^V(x) = 1.f(x) - 5.f(x+dx) + 10.f(x+2dx) - 10.f(x+3dx) + 5.f(x+4dx) - 1.f(x+5dx)$$

$$f'(x) = 5x^4 - 6x \quad f''(x) = 20x^3 - 6 \quad f'''(x) = 60x^2 \quad f^{IV}(x) = 120x \quad f^V(x) = 120$$

$$1.f(1) = +2, \quad -5.f(2) = -120, \quad 10.f(3) = +2200, \quad -10.f(4) = -14900$$

$$+5.f(5) = +15270, \quad -1.f(6) = -7672$$



5 points of cuts: 5 solutions.

Equations of degree 2 are solved by the formula:

$$\text{Ex: } ax^2 - bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{as you already know.}$$