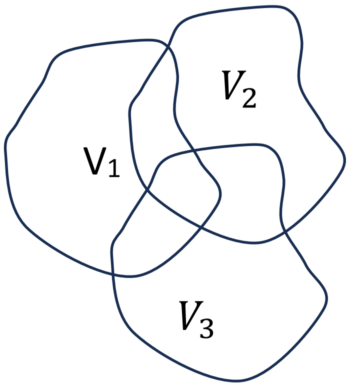
SET THEORY:



$V_1 \cup V_2 \cup V_3 - V_1 \cap V_2 - V_1 \cap V_3 - V_2 \cap V_3 -$

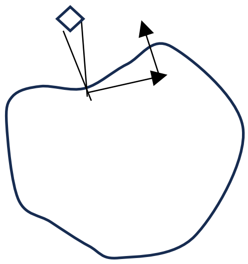
$V_1 \cap V_2 \cap V_3$

Once you know how to calculate length, now it's time for surface area and volume :

$V_m = \int_{\vec{m}} \sqrt{\left|\delta_{i,j}\right|}.dx_0.dx_1.dx_2.dx_3$ i,j = 0,1,2,3

$\vec{m}=\overrightarrow{dx_0}+\overrightarrow{dx_1}+\overrightarrow{dx_2}+\overrightarrow{dx_3}$ vectors unitary : $\widehat{u_0}, \widehat{u_1}, \widehat{u_2}, \widehat{u_3}$

AREA



and we will find the body area of 3-D. If we multiply it by one more term “h” , we will find the volume :

let's remember **arclength** when the triangle is not a rectangle :

u(t) iv(t)

L= $\int_a^b \sqrt{u'(t)^2\overrightarrow{r_u}\overrightarrow{r_u}+2u'(t)v'(t)\overrightarrow{r_u}\overrightarrow{r_v}+v'(t)^2\overrightarrow{r_v}\overrightarrow{r_v}}\, dt$

On **surfaces** :

we will call this one 3x3 matrix “M”.

where (a b c)= ($dx_0\ dx_1\ dx_2$)   where $\vec{n}=\overrightarrow{dx_0}+\overrightarrow{dx_1}+\overrightarrow{dx_2}$

and the vectors unit values of each $\overrightarrow{dx_n}$ will be: $\widehat{u_0}$, $\widehat{u_1}$,$\widehat{u_2}$

$$dS^3 = \begin{pmatrix} dx_0^{3/2} & dx_1^{3/2} & dx_2^{3/2} \end{pmatrix} . \text{"M"} . \begin{pmatrix} dx_0^{3/2} \\ dx_1^{3/2} \\ dx_2^{3/2} \end{pmatrix} \qquad \text{(i)}$$

the "M": $A_{000}=\widehat{u_0^3}$, $B_{001}= B_{100}= \widehat{u_0^2}\,\widehat{u_1}$, $C_{002}= C_{200}=\widehat{u_0^2}\,\widehat{u_2}$

$D_{101}=\widehat{u_0}\,\widehat{u_1^2}$, $E_{102}= E_{201}=\widehat{u_0}\;\widehat{u_1}\,\widehat{u_2}$, $F_{202}=\widehat{u_0}\,\widehat{u_2^2}$

Where $\widehat{u_0}$, $\widehat{u_1}$, $\widehat{u_2}$ do not form right angles with each other.

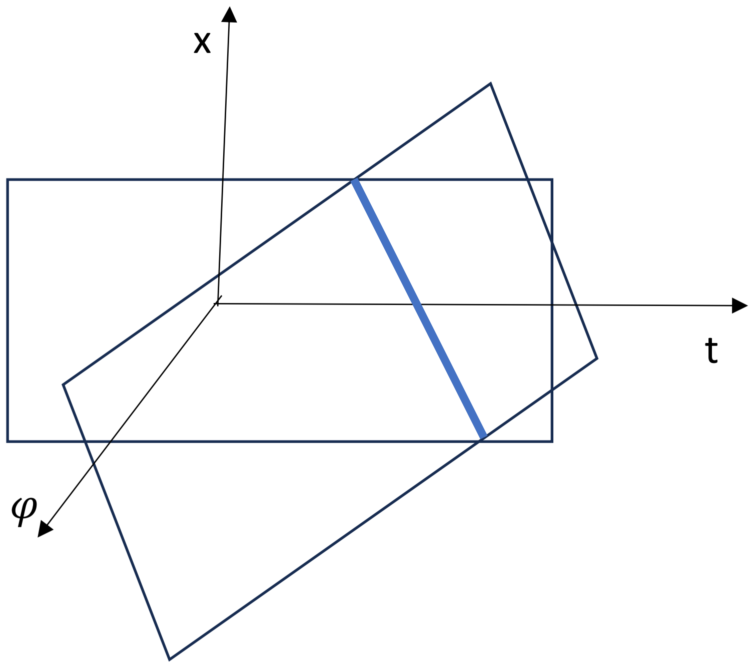
Let's assume that according to arclength : u(t)= x_0(t) , v(t)= x_1(t)

While for Surfaces we add another variable: x_2(t, φ)

Let's assume that an example : x_0 (t)= αt

x_1(t)= βt

x_2 (t,φ)$=$ γtφ



Obviously , the line between the planes is not perpendicular to "x" since x_2 depends on "t" and "φ".

$$M:\begin{bmatrix} A_{000} & B_{001} & C_{002} \\ B_{100} & D_{101} & E_{102} \\ C_{200} & E_{201} & F_{202} \end{bmatrix}$$

Now, operating algebraically (i), we will obtain :

$$dS^3 = dx_0^3\hat{u}_0^3 + 2dx_0^{3/2}dx_1^{3/2}\hat{u}_0^2\hat{u}_1 + 2dx_2^{3/2}dx_0^{3/2}\hat{u}_0^2\hat{u}_2 + 2dx_1^{3/2}dx_2^{3/2}\hat{u}_0\hat{u}_1\hat{u}_2 + dx_1^3\hat{u}_0\hat{u}_1^2 + dx_2^3\hat{u}_0\hat{u}_2^2$$

if now dS = $\sqrt[3]{(\dots)}.\frac{dt}{dt} \rightarrow \sqrt[3]{\frac{(\dots)}{dt^3}}.dt \rightarrow$

$$\sqrt[3]{\frac{dx_0^3\hat{u}_0^3}{dt^3} + 2\frac{dx_0^{3/2}}{dt^{3/2}}\frac{dx_1^{3/2}}{dt^{3/2}}\hat{u}_0^2\hat{u}_1 + 2\frac{dx_2^{3/2}}{dt^{3/2}}\frac{dx_0^{3/2}}{dt^{3/2}}\hat{u}_0^2\hat{u}_2 + + 2.\frac{dx_1^{3/2}}{dt^{3/2}}\frac{dx_2^{3/2}}{dt^{3/2}}\hat{u}_0\hat{u}_1\hat{u}_2 + \frac{dx_1^3}{dt^3}\hat{u}_0\hat{u}_1^2 + \frac{dx_2^3}{dt^3}\hat{u}_0\hat{u}_2^2}$$

In each term let's follow the next one mechanism :

Example: $\frac{[d(\alpha.t)]^3}{dt^3} = \left(\frac{d(\alpha.t)}{dt}\right)^3 = \alpha^3$,

$2.\left(\frac{d(\alpha.t)}{dt}\right)^{3/2}.\left(\frac{d(\beta.t)}{dt}\right)^{3/2} = \alpha^{3/2}\beta^{3/2}$ and so on until the end.

$$\alpha^3\hat{u}_0^3 + 2\alpha^{3/2}\beta^{3/2}\hat{u}_0^2\hat{u}_1 + 2\alpha^{3/2}(\gamma\varphi)^{3/2}\hat{u}_0^2\hat{u}_2 + 2\beta^{3/2}(\gamma\varphi)^{3/2}\hat{u}_0\hat{u}_1\hat{u}_2 + \beta^3\hat{u}_0\hat{u}_1^2 + (\gamma\varphi)^3\hat{u}_0\hat{u}_2^2$$

Let's put limits to "t" and " φ": $t_1 = 1, t_2 = 2, \varphi_1 = 1, \varphi_2 = 3$

S=$\int_{\varphi_1}^{\varphi_2}\left(\int_{t_1}^{t_2}\sqrt[3]{(\dots)}dt\right)d\varphi \rightarrow$S=$\int_{\varphi_1}^{\varphi_2}([\sqrt[3]{(\dots)}.t]_{t_1}^{t_2})d\varphi \rightarrow$

$\rightarrow S=\int_{\varphi_1}^{\varphi_2}\left(\sqrt[3]{(\dots)}\right)d\varphi \quad \rightarrow S=\int_{\varphi_1}^{\varphi_2}\left(\sqrt[3]{(a.\varphi^3+b.\varphi^{3/2}+c)}\right)d\varphi$

$a=\gamma^3$, $b= 2.(\alpha^{3/2}\gamma^{3/2}+\beta^{3/2}\gamma^{3/2})$,

$c=\alpha^3+2\alpha^{3/2}\beta^{3/2}+\beta^3=(\alpha^{3/2}+\beta^{3/2})^2$

$(\eta.\varphi^{3/2}+\varepsilon)^{2/3}=$ Newton binomial $=$

$=\binom{2/3}{0}.(\eta.\varphi^{\frac{3}{2}})^{2/3}\varepsilon^0+\binom{2/3}{1/3}.(\eta.\varphi^{\frac{3}{2}})^{1/3}\varepsilon^{1/3}+$
$\binom{2/3}{2/3}.(\eta.\varphi^{\frac{3}{2}})^0\varepsilon^{2/3}$

$$\binom{2/3}{0}=\frac{\left(\frac{2}{3}\right)!}{\left(\frac{2}{3}-0\right)!.0!}=\frac{\left(\frac{2}{3}\right)!}{.\left(\frac{2}{3}-\frac{2}{3}\right)!!.\left(\frac{2}{3}\right)!}\binom{2/3}{2/3}=1$$

While

$$\binom{2/3}{1/3}=\frac{\left(\frac{2}{3}\right)!}{\left(\frac{1}{3}\right)!.\left(\frac{1}{3}\right)!}=\frac{\Gamma(\frac{2}{3}+1)}{(\Gamma\left(1+\frac{1}{3}\right))(\Gamma\left(1+\frac{1}{3}\right))}=$$

$$\Gamma(x+1)=x!=x.\Gamma(x)$$

$$\Gamma(n+1/3)=\Gamma(1/3).\frac{(3n-2)!!!}{3^n}$$

$$\Gamma(1/3)=\Gamma(1-2/3)$$

$$\Gamma(1-2/3).\Gamma(2/3)=\frac{\pi}{Sin[\pi(\frac{2}{3})].}$$

$$\frac{\left(\frac{2}{3}\right).\Gamma\left(\frac{2}{3}\right).Sin^2\left(\frac{2\pi}{3}\right).(\Gamma\left(\frac{2}{3}\right))^2}{(1/9).\pi^2}=\frac{6}{\pi^2}.Sin^2\left(\frac{2\pi}{3}\right).(\Gamma\left(\frac{2}{3}\right))^3=\text{“C”}$$

$\left(a.\varphi^3 + b.\varphi^{\frac{3}{2}} + c\right) = (\eta.\varphi^{3/2} + \varepsilon)^2$

a=η^2 , b= 2.(η).ε , c=ε^2

S=$\int_{\varphi_1}^{\varphi_2} \left(\sqrt[3]{(\eta.\varphi^{3/2} + \varepsilon)^2}\right) d\varphi$

$(\eta.\varphi^{3/2}+\varepsilon)^{2/3} = 1.\eta^{2/3}.\varphi + "C".\varepsilon^{1/3}\eta^{1/3}\varphi^{1/2} + \varepsilon^{2/3}$

$(\eta.\varphi^{3/2} + \varepsilon)^2 = \eta^2\varphi + 2.\eta.\varepsilon.\varphi^{3/2} + \varepsilon^2$

S= $\int_{\varphi_1}^{\varphi_2}(\eta.\varphi^{3/2} + \varepsilon)^{2/3}\, d\varphi =$

$= \int_{\varphi_1}^{\varphi_2}(\eta^{2/3}.\varphi + "C".\varepsilon^{1/3}\eta^{1/3}\varphi^{1/2} + \varepsilon^{2/3})\, d\varphi =$

$\sqrt{a} = \eta \rightarrow \eta^{2/3} = \sqrt[3]{a}$

$\sqrt[3]{b} = (2^{1/3}\eta^{1/3}\varepsilon^{1/3})$ by chance $2^{1/3}$= “ C” ?

$\varepsilon^{2/3} = \sqrt[3]{c}$

S= $\int_{\varphi_1}^{\varphi_2}(\sqrt[3]{a}.\varphi + \sqrt[3]{b}.\varphi^{3/2} + \sqrt[3]{c})\, d\varphi =$

$= [\frac{\sqrt[3]{a}}{2}\varphi^2 + \frac{\sqrt[3]{b}}{5/2}\varphi^{5/2} + \sqrt[3]{c}\varphi]_1^3 = [\frac{\sqrt[3]{a}}{2}.9 + \frac{2\sqrt[3]{b}}{5}.\sqrt{3^5} + 3.\sqrt[3]{c}]$ -

$- [\frac{\sqrt[3]{a}}{2} + \frac{2\sqrt[3]{b}}{5} + \sqrt[3]{c}]$

Knowing that:

a=γ^3 , b= 2.($\alpha^{3/2}\gamma^{3/2} + \beta^{3/2}\gamma^{3/2}$) ,

c= $\alpha^3 + 2\alpha^{3/2}\beta^{3/2} + \beta^3 = (\alpha^{3/2} + \beta^{3/2})^2$

we can end up replacing “a, b, c”