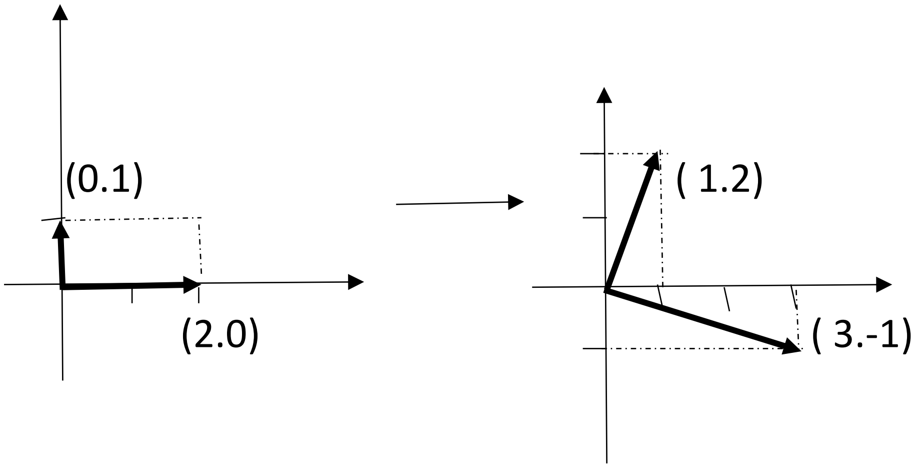
Determinants and Jacobian:



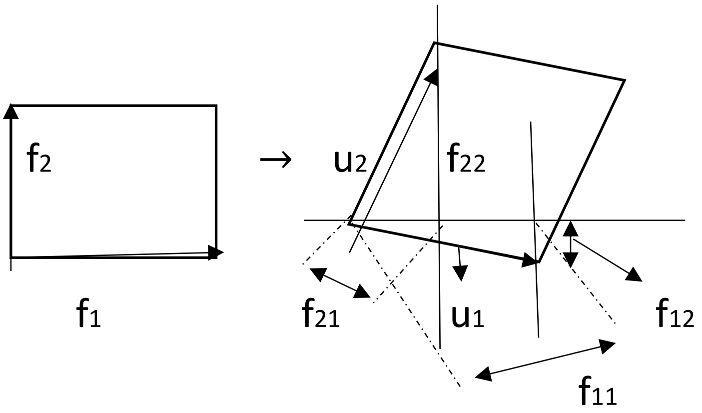
Then the matrix representing the change it is $\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$so that

$\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}3\\ -1\end{pmatrix}and\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}\begin{pmatrix}2\\ 0\end{pmatrix} = \begin{pmatrix}2\\ 4\end{pmatrix} = 2\begin{pmatrix}1\\ 2\end{pmatrix}$

$\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$is the Jacobian.

That is to say :

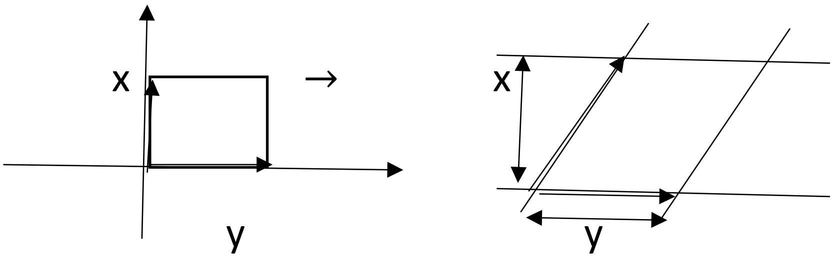
$f(f_1, f_2)$



area : $f_1 . f_2$

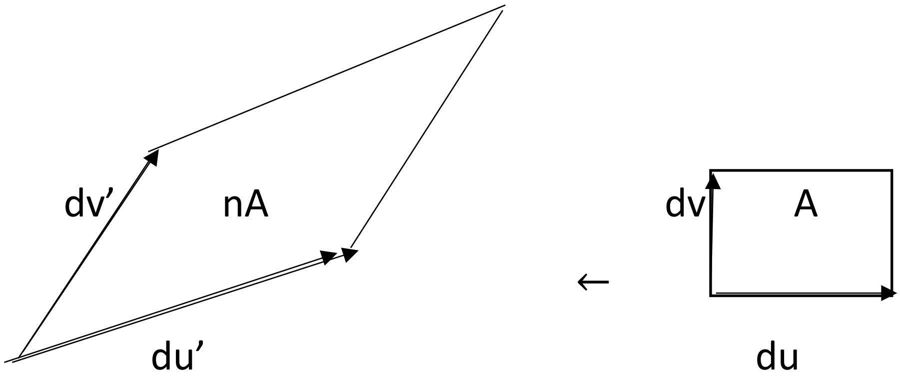
J(u,v)= $\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} . \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

Where



The area of the parallelogram remains constant since it is x.y

On the other hand



In Cartesian coordinates

$\iiint_{x,y,z} dV$while in polar:$\iiint_{r,\theta,\phi} r^2.Sin\theta dr d\theta d\phi$

Where dV = dxdydz J$(r, \theta, \phi) = r^2.Sin\theta$and

r $\geq$0, 0 $\leq \theta \leq \pi$, 0$\leq \phi \leq 2\pi$

The Jacobian represents how the area of the initial parallelogram is doubled .

Thus the Jacobian is the number of times the initial area:

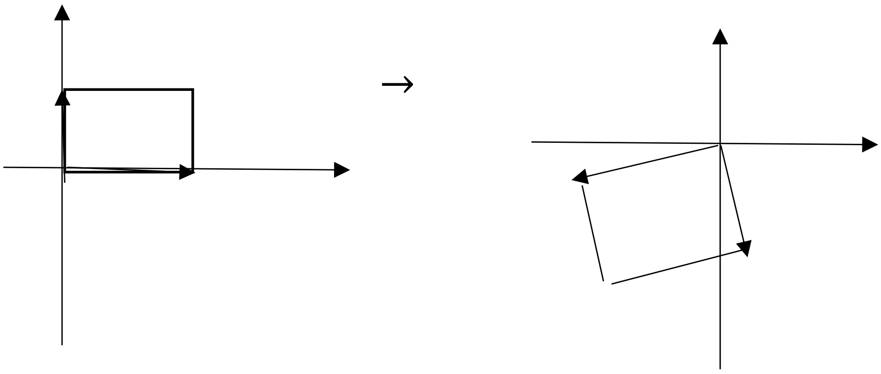
$\iint_{x,y} C.\grave{A}rea$where C is the Jacobian and is the determinant of

J(x,y).

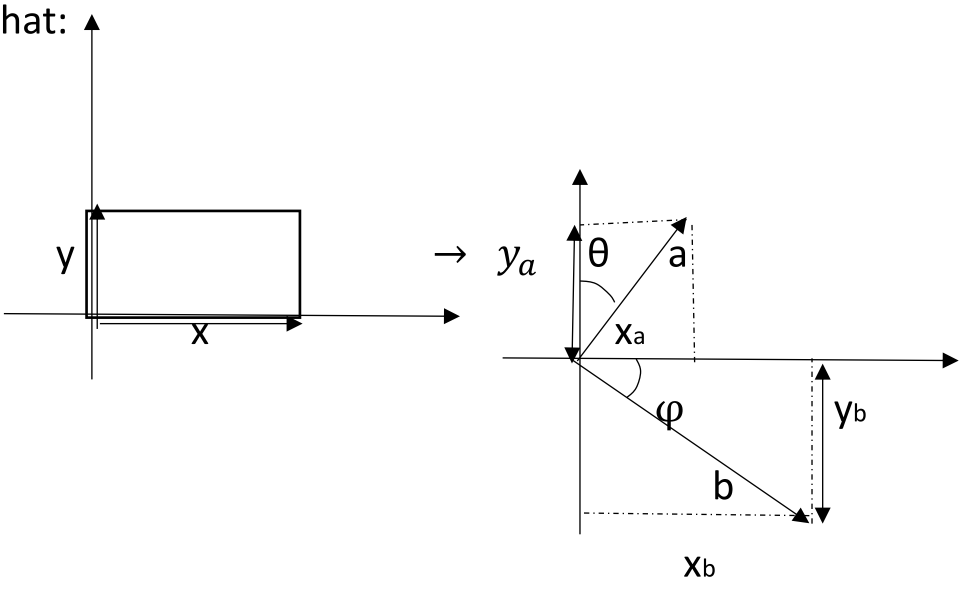
and the Area it is dxdy (or dxdydz in 3-D).

knowing that the Area encompassed is the determinant of the Jacobian .

The determinant can be negative if:



It is more , if we take into account the angles, us we will find with what:



x a = a.Sin θ , y a = a.Cos θ , x b = b.Cos φ , y b = - b.Sinφ

Jacobian (a,θ)= $\begin{pmatrix} x_a & x_\theta \\ y_a & y_\theta \end{pmatrix}$ JacobianJ (b, Φ)=$\begin{pmatrix} x_b & x_\Phi \\ y_b & y_\Phi \end{pmatrix}$

Jacobian (a,θ).$\begin{pmatrix} x_a \\ 0 \end{pmatrix} = \begin{pmatrix} x_a^2 \\ y_a.x_a \end{pmatrix}$, Jacobian (b,Φ).$\begin{pmatrix} x_b \\ 0 \end{pmatrix} = \begin{pmatrix} x_b^2 \\ y_b.x_b \end{pmatrix}$

Jacobian (a,θ).$\begin{pmatrix} 0 \\ y_a \end{pmatrix} = \begin{pmatrix} x_\theta \cdot y_a \\ y_\theta \cdot y_a \end{pmatrix}$, Jacobian (b,Φ).$\begin{pmatrix} 0 \\ y_b \end{pmatrix} = \begin{pmatrix} x_\theta \cdot y_b \\ y_\theta \cdot y_b \end{pmatrix}$

Finally , in the cases of integrals , dxdy , the Jacobian takes the form of derivatives:

J(r, θ)= $\begin{pmatrix} \partial x_r & \partial x_\theta \\ \partial y_r & \partial y_\theta \end{pmatrix}$

Also in 3-D: dxdydz
(Jacobian) will be a 3x3 matrix).