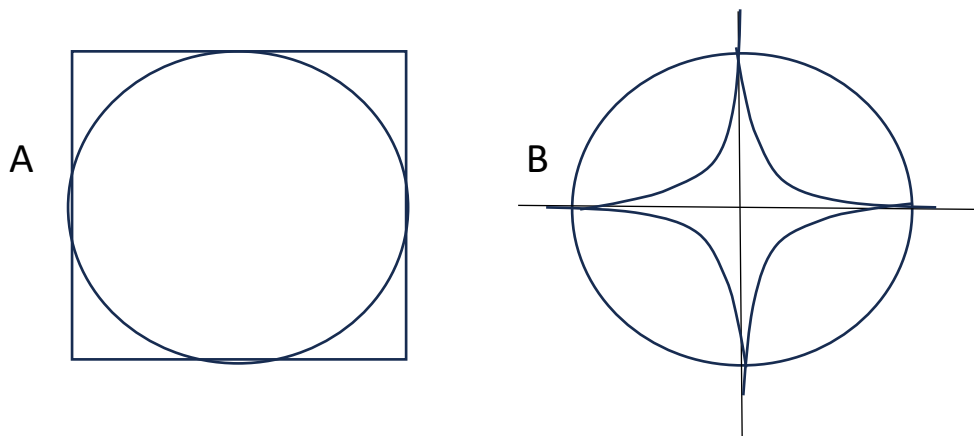
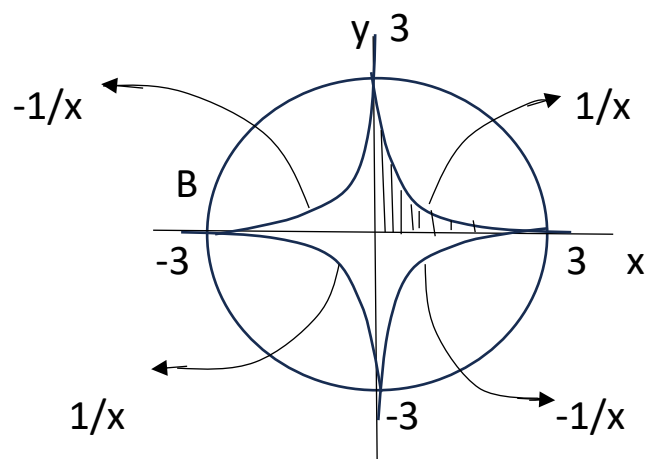


Area of the circular contour :

In the circle inscribed in a square (A) , let's fold the vertices none inside getting B



in B we can interpret the curves of each quadrant as a $y = 1/x$



We know that $x^2 + y^2 = R^2$ like this how when $x=1 \rightarrow y=1$

and when $x=-1 \rightarrow y=-1$

when $x=0 \rightarrow y=\infty$ and when $y=0 \rightarrow x=\infty$

suppose $R=3$ and $\lim_{x \rightarrow 0} y = 3$

to find the area pointed out we will do :

$$\int_0^3 \frac{1}{x} dx = \ln x \Big|_0^3 \rightarrow \ln 3 - \ln 0 \rightarrow \ln 3 - \ln(e^3)$$

let's put $y = \ln x \rightarrow x = e^y \rightarrow x = e^3$

instead when we treat $y = -1/x$: $\int_{-3}^0 -\frac{1}{x} dx = -\ln x \Big|_{-3}^0 \rightarrow$

$$\rightarrow -\ln(e^3) - (-\ln(-3)) \rightarrow -\ln(e^3) - (-\ln(0)) \rightarrow$$

$$\rightarrow -\ln(e^3) + \ln(-1) + \ln(3) \rightarrow -\ln(e^3) + i\pi + \ln(3)$$

Then add the areas : $\ln(3) - \ln(e^3) + (-\ln(e^3) + i\pi + \ln(3)) \rightarrow$

$\rightarrow$ and we multiply them by 2 since there are four quadrants:

$$\text{Area} = 2[2\ln(3) - 2\ln(e^3) + i\pi] = 4\ln(3) - 12 + 2i\pi$$

Note that also $\text{Area} = (2R)^2 - [\pi R^2]$