Binomials with exponent fractional II:

Knowing about the other acticle that (a+b) ^{0} = 1= C $_{0}$ i

(a+b) $^{0'5}$ oscillates between : $\frac{3}{2}a^{3/4}b^{3/4}$ and $\frac{3}{4}a^{3/4}+\frac{3}{4}b^{3/4}$ and the result is :

$$\frac{\frac{3}{2}a^{3/4}b^{3/4}+\frac{3}{4}a^{3/4}+\frac{3}{4}b^{3/4}}{2}=\frac{3}{4}a^{3/4}b^{3/4}+\frac{3}{8}a^{3/4}+\frac{3}{8}b^{3/4}= C_5$$

I want to elucidate (a+b) $^{1/3}$:

Exp : 0, 0'1, 0'2, 0'3, 0'4, 0'5→ 0, 1/10, 1/5, 1/3, 1/2'5, ½.

C $_{0}$ C $_{1}$ C $_{2}$ C $_{3}$ C $_{4}$ C $_{5}$

The value of C $_{3}$ ranges between 1 and C $_{5}$

We can resort to progressions geometric because the exponents increase progressively (as we see in the article BINOMIALS WITH FRACTIONAL EXPONENT I):

Let's remember :$(a+b)^n \rightarrow \sum exponents\ dels\ termes\ del\ binomi$
when n=0→ 1 , n=1→ 2 , n=2→ 6, n =3→ 12 , n=4 →20 …
so if n= 0'5 →1' 5 , n=1'5→ 4, n=2'5→ 9, n=3'5 →16
…

that is , they increase geometrically , therefore they follow this " series ":$a_n = a_k . r^{n-k}$

Like this :

$a_5 = a_1 . r^4$or whatever it is the same : $C_5 = C_0 . r^5$= $1. r^5$

$$C_5 = (a+b)^{0'5} = \frac{\left[\left(\frac{3}{2}\right).a^{3/4}.b^{3/4}\right]+\left[\left(\frac{3}{4}\right).a^{\frac{3}{4}}+\left(\frac{3}{4}\right).b^{3/4}\right]}{2}$$

r=$\sqrt[5]{\frac{3}{4}a^{3/4}b^{3/4}+\frac{3}{8}a^{3/4}+\frac{3}{8}b^{3/4}}$

C 5 is composed of three terms, then can assimilate to

(" A"+ "B") ^{2} = "A" ^{2} + 2 ."A " ."B "+ "B" ^{2}

We deduce : (" A"+ "B") ^{2} = ($\sqrt{\frac{3}{8}}a^{\frac{\sqrt{3}}{2}}+\sqrt{\frac{3}{8}}b^{\frac{\sqrt{3}}{2}}$) ^{2}

Thus : $\sqrt[5]{(\sqrt{\frac{3}{8}}a^{\frac{\sqrt{3}}{2}}+\sqrt{\frac{3}{8}}b^{\frac{\sqrt{3}}{2}})^2}$= r

therefore : C 3 = = $C_0.r^3$=1. [$\sqrt[5]{(\sqrt{\frac{3}{8}}a^{\frac{\sqrt{3}}{2}}+\sqrt{\frac{3}{8}}b^{\frac{\sqrt{3}}{2}})^2}$]^3=

$=(\sqrt{\frac{3}{8}}a^{\frac{\sqrt{3}}{2}}+\sqrt{\frac{3}{8}}b^{\frac{\sqrt{3}}{2}})^{6/5}$

And I 've come this far .

$\boxed{(a+b)^{1'5} = C_{15}}$ C_{10} C_{11} C_{12} C_{13} C_{14} C_{15}

(a+b)^1 (a+b)$^{1'1}$ … (a+b)$^{1'5}$

According to BINOMIALS WITH FRACTIONAL EXPONENT I:

C 15 = ½.([($\frac{3}{2}$). $a^{3/2}b^{1/2}$ + +($\frac{3}{2}$). $a^{1/2}b^{3/2}$]+ +[($\frac{3}{4}$). $a^{4/3}$+ ($\frac{3}{2}$). $a^{2/3}b^{2/3}$+ ($\frac{3}{4}$). $b^{4/3}$])

C 15 =C 10 . r ^{5} r=$\sqrt[5]{\frac{C_{15}}{(a+b)}}$

Then : $r^5=\left(\frac{3}{4}\right).\frac{a^{3/2}.b^{1/2}}{(a+b)}+\left(\frac{3}{4}\right).\frac{a^{1/2}.b^{3/2}}{(a+b)}+$

$+\left(\frac{3}{8}\right).\frac{a^{4/3}}{(a+b)}+\left(\frac{3}{4}\right).\frac{a^{2/3}.b^{2/3}}{(a+b)}+\left(\frac{3}{8}\right).\frac{b^{4/3}}{(a+b)}$

Attention :

(a+ b) $^{4/3}$ = (a+ b) $^{1'333}$ =

= α. $a^{4/3}b^0$+β. $a^1b^{1/3}$+γ. $a^{2/3}b^{2/3}$+δ. $a^{1/3}b^1$+ε. $b^{4/3}a^0$

When (4/ 3)/ 4= 1/3 what you want say that we will divide it $\sqrt[3]{(a+b)^4}$into 5 terms (according to Newton's binomial and Tartaglia 's triangle) :

So that :

α= $\binom{4/3}{4/3}$ β = $\binom{4/3}{1}$ γ = $\binom{4/3}{2/3}$ δ= $\binom{4/3}{1/3}$ ε = $\binom{4/3}{0}$

β= $\binom{4/3}{1}$= $\frac{(4/3)!}{1!.\left(\frac{4}{3}-1\right)!}$= $\frac{\left(\frac{4}{3}\right).\Gamma(1+\frac{1}{3})}{.\Gamma(\frac{1}{3})}$= $\frac{\left(\frac{4}{3}\right).\left(\frac{1}{3}\right).\Gamma(\frac{1}{3})}{.\Gamma(\frac{1}{3})}$= 4/9

because we know that :. $\Gamma\left(n+\frac{1}{3}\right)=\Gamma\left(\frac{1}{3}\right).\frac{(3n-2)!!!}{3^n}$

when n= 1

(3n-2)!!!= (3n-2).((3n-2)-3).((3n-2)-5).((3n-2)- 8)...

γ = $\binom{4/3}{2/3}$= $\frac{(4/3)!}{\left(\frac{2}{3}\right)!\left(\frac{2}{3}\right)!}$= $\frac{\left(\frac{4}{3}\right).\left(\frac{1}{3}\right).\Gamma(\frac{1}{3})}{(\frac{2}{3})^2(\Gamma\left(\frac{2}{3}\right))^2}$=$\frac{\pi}{Sin(\frac{\pi}{3})(\Gamma\left(\frac{2}{3}\right))^2}$

knowing that: $\Gamma\left(\frac{1}{3}\right).\Gamma(1-\frac{1}{3})$=$\frac{\pi}{Sin(\frac{\pi}{3})}$

and δ= $\binom{4/3}{1/3}$= $\frac{(4/3)!}{\left(\frac{1}{3}\right)!.1!}$= $\frac{\left(\frac{4}{3}\right).\left(\frac{1}{3}\right).\Gamma(\frac{1}{3})}{\Gamma(1+\frac{1}{3})}$= 4/3

there is nothing more see what $\Gamma(n)$it has values as long as n$\neq 0$ nor n $\neq$negative integers. So $\Gamma(1/2)$=$\sqrt{\pi}$ $\Gamma(-1/2)$= - 2$\sqrt{\pi}$
$\Gamma(1/3)$= ∃, $\Gamma(-3/2)$= ∃, $\Gamma(-7/2)$= ∃, $\Gamma(2)$=∃

$\Gamma(-3)$= ∄... etc. ...

SEDIMENTING KNOWLEDGE:

1) Progression Geometric : $C_n = C_m \cdot r^{n-m}$

2) Binomials with exponent fractional : $(a+ b)^{n/m}$ where n>m

3) Binomials with exponent fractional but $(a+ b)^{n/m}$ where n<m

4) Finally for Newton's binomial and the Triangle of Tartàglia :

$(a+ b)^{p} = \alpha\, a^p b^0 + \beta\, a^{p-1} b^1 + \gamma\, a^{p-2} b^2 + \cdots + \delta a^0 b^p$

as long as p is no. integer positive

and in the example $C_3 = (\sqrt{\frac{3}{8}} a^{\frac{\sqrt{3}}{2}} + \sqrt{\frac{3}{8}} b^{\frac{\sqrt{3}}{2}})^{6/5}$

we see that (6/ 5)/ 6= 1/ 5 , therefore :

$(\text{“ X”+ ”Y”})^{6/5} = (\text{“ X”+ ”Y”})^{1'2}$

$$= \binom{6/5}{6/5} x^{6/5} Y^0 + \binom{6/5}{1} X^1 Y^{1/5} + \binom{6/5}{4/5} X^{4/5} Y^{2/5} + \binom{6/5}{3/5} X^{3/5} Y^{3/5} + \binom{6/5}{2/5} X^{2/5} Y^{4/5} + \binom{6/5}{1/5} X^{1/5} Y^1 + \binom{6/5}{0} X^0 Y^{6/5}$$