

Case of the Laplace Transform:

$$\int \{t^a\} = \int_0^{\infty} t^a \cdot e^{-st} \cdot dt = t^a \cdot \frac{e^{-st}}{-s} \Big|_0^{\infty} - \int_0^{\infty} a \cdot \frac{1}{-s} t^{a-1} \cdot e^{-st} \cdot dt =$$

$$u_1 = t^a$$

$$du_1 = a t^{a-1} \cdot dt$$

$$dv_1 = e^{-st} \cdot dt$$

$$v_1 = \frac{e^{-st}}{-s}$$

$$= t^a \cdot \frac{e^{-st}}{-s} \Big|_0^{\infty} + \left(\frac{a}{s} \cdot \Gamma(a) \right)$$

$$\frac{1}{s} \cdot a \cdot \Gamma(a) = \frac{1}{s} \cdot \Gamma(a+1)$$

Assuming that $0 \cdot \infty \rightarrow 0$

$$\frac{a}{s} \cdot t^{a-1} \cdot \frac{e^{-st}}{-s} \Big|_0^{\infty} - \frac{a}{s} \int_0^{\infty} (a-1) \cdot \frac{1}{-s} t^{a-2} \cdot e^{-st} \cdot dt =$$

also assume that = 0

$$u_2 = t^{a-1}$$

$$du_2 = (a-1) \cdot t^{a-2} \cdot dt$$

$$\frac{1}{s^2} \cdot a \cdot (a-1) \cdot \Gamma(a-1)$$

$$dv_2 = e^{-st} \cdot dt \quad v_2 = \frac{e^{-st}}{-s}$$

and if we apply the integral by parts again :

$$0 - \frac{a}{s^2} \int_0^{\infty} (a-1)(a-2) \cdot t^{a-3} \cdot \frac{1}{-s} e^{-st} \cdot dt =$$

$$= \frac{a \cdot (a-1) \cdot (a-2)}{s^3} \cdot \Gamma(a-2)$$

and in another step , $\frac{a \cdot (a-1) \cdot (a-2) \cdot (a-3)}{s^4} \cdot \Gamma(a-3)$ etc. ...

Knowing that: $\Gamma(a+2) = (a+1) \cdot \Gamma(a+1) = (a+1) \cdot a \cdot \Gamma(a)$

and in general: $\Gamma(a+n) = (a+(n-1)) \dots (a+(n-2)) \dots (a) \cdot \Gamma(a)$

$$\text{and: } \int \{t^a\} = \frac{a \cdot (a-1) \cdot (a-2) \dots (a-n)}{s^{n+1}} \cdot \Gamma(a-n)$$