Feynman integral:

$\int_0^{\pi/4} \frac{1}{Cos^a x} dx = I(a)$

I'(a)=$\int_0^{\pi/4} \frac{d}{da}(Cos^{-a}x)dx = \int_0^{\pi/4} (Cos^{-a}x).Ln(Cosx).(-1)dx$

Knowing that y= a^x y'= a^x.lna.dx

Suppose m= $Cos^{-1}x$= 1/Cosx and Ln($Cos^{-1}x$)= - Ln(Cosx)

Then : $\int m^a.[-Ln(m)].(-dx)$= I(a).Ln (m)

I'(a)= I(a). Ln (m) →differential equation fulfilled by the function "e^x".

Solution : I(a)=$e^{Ln(m).a}$

and now I (a) | $^{\pi/4}_{0}$= $e^{Ln(Cos(\pi/4).a}$- $e^{Ln(Cos(0).a}$= $e^{Ln\left(\frac{\sqrt{2}}{2}\right).a}$-1