

## DIGAMMA FUNCTION and CTNT EULER-MASCHERONI:

$$\Gamma(s+2) = (s+1) \cdot (s) \cdot \Gamma(s)$$

$$\text{function : } \Psi(s) = \frac{\Gamma'(s)}{\Gamma(s)}$$

$$\Psi(s+1) = \frac{\Gamma'(s+1)}{\Gamma(s+1)} = \frac{s \cdot \Gamma'(s) + \Gamma(s)}{s \cdot \Gamma(s)} = \Psi(s) + \frac{1}{s}$$

$$\Psi(s+1) = -\gamma + \sum_{l=1}^{\infty} \left( \frac{1}{l} - \frac{1}{l+s} \right)$$

$$\Gamma'(s) = \left[ \Psi(s+1) - \frac{1}{s} \right] \cdot \Gamma(s)$$

$$\frac{d}{ds} (s+1)! = [2s+1] \cdot \Gamma(s) + [s \cdot (s+1)] \cdot \Gamma'(s) =$$

$$= (2s+1) \cdot \Gamma(s) + [s \cdot (s+1)] \cdot \left[ \Psi(s+1) - \frac{1}{s} \right] \cdot \Gamma(s) =$$

$$= (2s+1) \cdot \Gamma(s) + [s \cdot (s+1)] \cdot \left( -\frac{1}{s} \cdot \Gamma(s) + \Gamma(s) \cdot \Psi(s+1) \right) =$$

$$= \Gamma(s) [(2s+1) - (s+1) + \Psi(s+1)]$$

$$\Gamma(0), \Gamma(-1), \Gamma(-2) \dots \nexists \quad \Gamma'(1) = -\gamma$$

$$\Psi(s) = -\frac{1}{s} - \gamma + \sum_{m=1}^{\infty} \left( \frac{1}{m} - \frac{1}{m+s} \right)$$

$$\Gamma'(s) = (s-1)! \cdot [H_{s-1} - \gamma] = s! \cdot [H_s - \gamma]$$

While by definition :

$$\gamma = \lim_{m \rightarrow \infty} (H_m - \ln(m))$$

$$\text{And the series Harmonica it is } H_s = \sum_{s=1}^s \left( \frac{1}{s} \right)$$

In other words, including the works by Armengol Gasull, we see the gamma constant  $\gamma$  in another perspective

"n" measurements  $\rightarrow$  "n" results:  $x_1, x_2, x_3, x_i \dots x_n$

$$E(Y_i) = x_i \cdot p_i + 0 \cdot (1 - p_i) = x_i \cdot p_i = x \text{ and } (1/i),$$

in the case "i=2" and assuming that  $x_i=1$  (and so on from "1" to "n")

We measure the total probability density:  $S(p_{\text{total}})$  or Shannon entropy:

$$S(p_{\text{total}}) = - \sum_{i=1}^n p_i \cdot \ln(p_i) = - \sum_{i=1}^m \frac{1}{n_i} \ln n_i$$

Knowing that  $\sum_{n=1}^m E(Y_n) = x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 p_3 + \dots + x_m \cdot p_m$

We define, as we have already done, studied previously, that

$$\lim_{m \rightarrow \infty} (\sum_{n=1}^m H_n - \ln(m)) = \gamma$$

$$S(p_{\text{total}}) = - \sum_{i=1}^n p_i \cdot \ln(p_i) = - \sum_{i=1}^m \frac{1}{n_i} \ln n_i$$

$$p_i = 1/n_i$$

Taking out logarithms to  $E(Y_i) = x_i \cdot p_i$  we will obtain

$$\ln[E(Y_i)] = \ln(x_i) + \ln(p_i)$$

which, deriving, we obtain

$$\frac{d}{dx} \ln[E(Y_i)] = \frac{1}{x_i} dx + \frac{d}{dx} \ln\left(\frac{1}{n_i}\right) = \frac{1}{x_i} dx + \frac{d}{dx} (-\ln(n_i))$$

if we now integrate again:

$$\ln[E(Y_i)] = \int_1^m \frac{1}{x_i} - \ln(m_i)$$

$$\ln[E(Y_i)] = \lim_{m \rightarrow \infty} \left( \sum_{i=1}^m \frac{1}{x_i} - \ln(m_i) \right)$$