How to solve equations with cubic, quartic, quintic roots II:

Let's give an example:

$x^5 + 4x^3 - x^2 + 3 = 0$

suppose $x^5 = +a \rightarrow x = \sqrt[5]{a} \rightarrow \frac{x}{\sqrt[5]{a}} = 1$

$4x^3 - x^2 + 3 = -x^5 \rightarrow \frac{4x^3 - x^2 + 3}{-a} = 1 = \frac{x}{\sqrt[5]{a}} \rightarrow$

$\sqrt[5]{a}.(4x^3 - x^2 + 3) = -ax \rightarrow x = \frac{-\sqrt[5]{a}}{a}.(4x^3 - x^2 + 3)$

$(4x^3 - x^2 + 3) = -\sqrt[5]{a^4}.x$

Now $x^3 = +b \rightarrow \frac{x}{\sqrt[3]{b}} = 1 \rightarrow 4.b = -\sqrt[5]{a^4}.x + x^2 - 3 \rightarrow$

$\frac{-\sqrt[5]{a^4}.x + x^2 - 3}{4b} = 1 = \frac{x}{\sqrt[3]{b}} \rightarrow \frac{\sqrt[3]{b}}{b}(-\sqrt[5]{a^4}.x + x^2 - 3) = 4x \rightarrow$

$(-\sqrt[5]{a^4}.x + x^2 - 3) = 4x.\sqrt[3]{b^2} \rightarrow x2 = 4.\sqrt[3]{b^2}.x + \sqrt[5]{a^4}.x + 3 =$

$$x^2 = +c \rightarrow (x/\sqrt{c}) = 1$$

$\rightarrow \frac{4.\sqrt[3]{b^2}.x + \sqrt[5]{a^4}.x + 3}{c} = 1 = \frac{x}{\sqrt{c}} \rightarrow x = \sqrt{c}.\frac{4.\sqrt[3]{b^2}.x + \sqrt[5]{a^4}.x + 3}{c} \rightarrow$

$x(\sqrt{c}) = 4.\sqrt[3]{b^2}.x + \sqrt[5]{a^4}.x + 3 \rightarrow x(4.\sqrt[3]{b^2} + \sqrt[5]{a^4} - \sqrt{c}) = -3 \rightarrow$

$\rightarrow x = \frac{-3}{4.\sqrt[3]{b^2} + \sqrt[5]{a^4} - \sqrt{c}}$

$\sqrt[5]{a^4} \rightarrow \sqrt[5]{a^2}.\sqrt[5]{a^2} \rightarrow$ (+ +) $^{1/5}$, (+ +) $^{1/5}$ A

ó (- +) $^{1/5}$, (- +) $^{1/5}$ A'

$\sqrt[4]{b^3} \rightarrow \sqrt[4]{b}.\sqrt[4]{b^2} \rightarrow \sqrt{d}$

$\underbrace{(+)^{1/4}}_{C}$, $\underbrace{(-)^{1/4}}_{C'}$

$\sqrt[3]{c^2} \rightarrow \underbrace{(+\,+)^{1/3}}_{D}$ ó $\underbrace{(-\,+)^{1/3}}_{D'}$

$\sqrt[2]{d} \rightarrow \underbrace{(+)^{1/2}}_{E}$, $\underbrace{(-)^{1/2}}_{E} \rightarrow$ "i"

'

The problem lies in whether the roots must only be real or whether the imaginary ones are also included.

Please note that I always assume that x^n = + number and not

x^n =– number.

Another example:

$2x^4 + x^2 - 3x+4 = 0 \rightarrow x^2 - 3x+4 = -2x^4 \rightarrow \frac{x^2 - 3x+4}{-2a} = 1 = \frac{x}{\sqrt[4]{a}} \rightarrow$

$x^4 = +ax = \sqrt[4]{a} \quad 1 = \frac{x}{\sqrt[4]{a}}$

$\rightarrow -\frac{2ax}{\sqrt[4]{a}} = x^2 - 3x+4 \rightarrow -2.\sqrt[4]{a^3}.x + 3x - 4 = x^2 \rightarrow$

$x^2 = +bx = \sqrt{b} \quad \frac{x}{\sqrt{b}} = 1$

$\rightarrow$doing the same procedure of equalizing to 1:

$\frac{-2.\sqrt[4]{a^3}.x + 3x - 4}{b} = 1 = \frac{x}{\sqrt{b}}$ $\rightarrow$gives: $-2.\sqrt[4]{a^3}.x + 3x - 4 = \frac{b.x}{\sqrt{b}} \rightarrow$

$\rightarrow -2.\sqrt[4]{a^3}.x + 3x - 4 - \sqrt{b}.x = 0 \rightarrow x(-2\sqrt[4]{a^3} - \sqrt{b} + 3) = +4$

$\rightarrow x = \frac{4}{\left(-2\sqrt[4]{a^3} - \sqrt{b} + 3\right)}$

$\sqrt[4]{a^3} \rightarrow \sqrt[4]{b}.\sqrt[4]{b^2}$according to previous reasoning C, C'

$\sqrt[2]{b}$ $\rightarrow$according to previous reasoning E, E'

Adding them all together gives 4 solutions!!

ANOTHER EXAMPLE:

$x^3 + x^2$-2= 0

We will do the same process, but since it is cumbersome I will get to the point:

x ^{3} = a ... x= $\sqrt[3]{a}$.x ... $\frac{a.x}{\sqrt[3]{a}}$= 2-x ^{2} ... $-\sqrt[3]{a^2}.x - 2$= x ^{2} = b ...

$\frac{-\sqrt[3]{a^2}.x-2}{b}$= 1= $\frac{x}{\sqrt{b}}$... $-\sqrt[3]{a^2}.x - 2$= x. $\sqrt{b}$... x= $\frac{-2}{\sqrt[3]{a^2}+\sqrt{b}}$...

Where it $\sqrt[3]{a^2}$offers →(+ +) $^{1/3}$ or (+ -) $^{1/3}$

$\sqrt{b}$ →(+) $^{1/2}$ = 1 , (-) $^{1/2}$ = i

It agrees with the number of roots of the polynomial: 3.

And now I will propose a new equation:

$x^6 = 1$

I can also express x^6 as ~ $\sqrt[7]{x^7\sqrt[6]{x^6\sqrt[5]{x^5\sqrt[4]{x^4\sqrt[3]{x^3\sqrt{x^2\sqrt{x}}}}}}}$= +1

$(-1)^7(-1)^6(-1)^5(-1)^4(-1)^3(-1)^2(-1)^1$ →7 negative values, from which +1 is obtained

$(+1)^7(+1)^6(+1)^5(+1)^4(+1)^3(+1)^2(+1)^1$ →no negative value: =+1

$(-1)^7(-1)^6(-1)^5(-1)^4(-1)^3(\pm1)^2(-1)^1$ →6 negative values and one positive. As long as they are not these 4: x^7neither x^5nor x^3nor x [which being odd $(-1)^7 = -1$, $(-1)^5 = -1$,

$(-1)^3 = -1$, x= -1, which being 4 end up giving: +1].

The terms x^2, x^4, $x^6$are the same if x= +1 or x= -1.

5 negative values: for example:

$(-1)^7(-1)^6(-1)^5(\pm1)^4(-1)^3(\pm1)^2(-1)^1$ →is similar to the case of 6

negative values: the terms x^2, x^4, x^6it doesn't matter if = +1

or = -1

$(-1)^7(\pm1)^6(-1)^5(-1)^4(-1)^3(\pm1)^2(-1)^1$

$(-1)^7(\pm1)^6(-1)^5(\pm1)^4(-1)^3(-1)^2(-1)^1$

4 negative values: it is the maximum of "-1" that must be there to achieve $x^6 = 1$. the 4 odd ones are canceled and it only has one expression: $(-1)^7(\pm1)^6(-1)^5(\pm1)^4(-1)^3(\pm1)^2(-1)^1$

While we can also include the option 2 negative values $(+1)^7(\pm1)^6(+1)^5(\pm1)^4(-1)^3(\pm1)^2(-1)^1$ →the 2 negative values, as we can intuit, can only be located at x^7, x^5, x^3, x with 2 combination possibilities:

$(+1)^7(\pm1)^6(+1)^5(\pm1)^4(-1)^3(\pm1)^2(-1)^1$

$(-1)^7(\pm1)^6(+1)^5(\pm1)^4(+1)^3(\pm1)^2(-1)^1$

$(+1)^7(\pm1)^6(-1)^5(\pm1)^4(+1)^3(\pm1)^2(-1)^1$

$(+1)^7(\pm1)^6(-1)^5(\pm1)^4(-1)^3(\pm1)^2(+1)^1$

$(-1)^7(\pm1)^6(-1)^5(\pm1)^4(+1)^3(\pm1)^2(+1)^1$

$(-1)^7(\pm1)^6(+1)^5(\pm1)^4(-1)^3(\pm1)^2(+1)^1$

The case of a negative value is not allowed since it could result in a value of -1 and would no longer be a root of $x^6 = +1$.

and that's it.

and to make the mechanism I use clearer, I will propose another example:

$x^5 + 2x^4 - x^2 + 4 = 0$ $\quad x^5 = +a \quad x^4 = +b \quad x^2 = +c$

x= f($\sqrt[5]{a^4}$, $\sqrt[4]{b^3}$, $\sqrt[2]{c}$, C) where C ≡4 →x= $\frac{-4}{\sqrt[5]{a^4}+2\sqrt[4]{b^3}-\sqrt[2]{c}}$

solutions: A or A’, C and C’ , E and E’.

equation: $\quad x^5 + 2x^4 - x^2 + 4 = 0$

solutions: $\quad 1 \quad 2 \quad 2$

And counting everything gives 5 solutions!!!

$x^5 = \sqrt[2]{x^6 \sqrt[2]{x^5 \sqrt[2]{x^4 \sqrt[2]{x^3 \sqrt[2]{x^2 \sqrt[2]{x}}}}}} = +a$

when a= 1, n= 1, 6

1. Case in which all x^n's are "+ ".

Case in which we only have 1 x= "- " ∄, since it is not possible to replace all the x n ' s so that it is = +a

2. Case in which we have 2 x= "- ":

(+1, +1, +1, -1, +1, -1)

(+1, -1, +1, +1, +1, -1)

(+1, -1, +1, -1, +1, +1)

3. Case in which we have 3 x= "- ":

(-1, -1, +1, -1, +1, +1) (+1, -1, -1, -1, +1, +1)

(-1, +1, +1, -1, +1, -1) (+1, -1, +1, -1, -1, +1)

(-1, -1, +1, +1, +1, -1) (+1, +1, -1, -1, +1, -1)

(+1, +1, +1, -1, -1, -1) (+1, -1, +1, +1, -1, -1)

(+1, -1, -1, +1, +1, -1)

4. Case in which we have 4 x= "- ":

(also possible)

5. Case in which we have 5 x= "- ":

(-1, -1, -1, -1, -1, +1)

(-1, +1, -1, -1, -1, -1)

(-1, -1, -1, +1, -1, -1)

And all five cases together result in the 5 possible combinations: the 5 roots of "$x^5 = +1$".

$x^5 + x^4 + x^3 + x^2$ +x= 3

$$x=\frac{3}{\underset{\substack{\downarrow\\ 1}}{\sqrt[5]{a^4}}+\underset{\substack{\downarrow\\ 2}}{\sqrt[4]{b^3}}+\underset{\substack{\downarrow\\ 1}}{\sqrt[3]{c^2}}+\underset{\substack{\downarrow\\ 1}}{\sqrt[2]{d}}+1}$$

1 2 1 1 → 5 solutions

Conclusion: $\sqrt[4]{b^2} = \sqrt[2]{b} = \sqrt[2]{d}$

$\sqrt[2]{d}$ it has an imaginary root and a real root: E and E'.

In all equations there are imaginary roots such as C' and E': $(-)^{1/4}$ and $(-)^{½}$ on the other hand, E and C are equivalents.

Example :

$x^5 + x^3 = 1 \rightarrow$ -1+ $x^3 = - x^5$

x ^{5} = -a $\rightarrow$1=$\frac{x}{\sqrt[5]{(-1).a}}$

-1+ x ^{3} = a$\rightarrow$ $\frac{-1+x^3}{a}$= 1=$\frac{x}{\sqrt[5]{(-1).a}}$ $\rightarrow$ -1+ x ^{3} = x.$\frac{a}{\sqrt[5]{(-1).a}}$ $\rightarrow$

-1+ x ^{3} = x. $\frac{a}{\sqrt[5]{(-1)}\sqrt[5]{a}}$= x.$\frac{\sqrt[5]{a^4}}{\sqrt[5]{(-1)}}$

while = $\sqrt[5]{(-1)}$= $\sqrt[10]{(-1)^2}$= $\sqrt[10]{(1)}$ =1.

x^3= -b x= f($\sqrt[5]{a^4}$, $\sqrt[3]{b^2}$, -1) x=$\frac{1}{\sqrt[5]{a^4}+\sqrt[3]{b^2}}$

But… be carefull because x^3 =$x_1.x_2.x_3 = \frac{1}{x^2+1}$

Those x^2 are the two roots that lacked appart from A or A' and D or D' , (described in x_1, x_2 and x_3).

or in the case of $x^4 - x^3 = -2$,

x ^{4} = - b

… 1= $\frac{x}{\sqrt[4]{(-1).b}}$… -x ^{3} + 2= -x ^{4} … $\frac{-x^3+2}{b}$= 1= x. $\frac{1}{\sqrt[4]{(-1).b}}$

$\rightarrow$ $-x^3 +$ 2= x.$\frac{\sqrt[4]{b^3}}{\sqrt[4]{(-1)}}$ where $\sqrt[4]{(-1)}$= $\sqrt[8]{(-1)^2}$= 1

And so with x ^{3} = -c x= f($\sqrt[4]{b^3}$, $\sqrt[3]{c^2}$,2) $x=\frac{-2}{\sqrt[4]{b^3}+\sqrt[3]{c^2}}$

In this way, it works either if we use x ^{n} = +p or x ^{n} = -p