STIRLING APPROXIMATION.

LnN !~ N.lnN - N

P(x)= e^{x}= 1+x.f'(a)+ $\frac{x^2.f''(a)}{2!}+\frac{x^3.f'''(a)}{3!}+\frac{x^4.f^{IV}(a)}{4!}$ +…=

=$\sum_{n=0}^{\infty}(+1)^n.\frac{x^n}{n!}.f^n(a)$

P(x)= e^{-x}= 1- x.f'(a)+ $\frac{x^2.f''(a)}{2!}-\frac{x^3.f'''(a)}{3!}+\frac{x^4.f^{IV}(a)}{4!}$ +…=

=$\sum_{n=0}^{\infty}(-1)^n.\frac{x^n}{n!}.f^n(a)$

On the other hand, P(x)=$\sum_{n=0}^{\infty}a_n.x^n$

$\int_0^\infty x^n . e^{-x} . dx$=

$x^n = t \to \quad Ln(x^n) = Lnt \;\to\; x = (e^{Lnt})^{\frac{1}{n}} \to$

$x = e^{(Lnt)/n}$ →dx= $e^{(Lnt)/n} . \frac{1}{t^{1/n}} . \frac{t^{(\frac{1}{n}-1)}}{n} . dt$

$\int_0^\infty e^{Lnt} . e^{-t^{1/n}} . e^{Lnt^{1/n}} . \frac{1}{n.t}$ dt = $\int_0^\infty e^{Lnt+Lnt^{1/n}-t^{1/n}} . \frac{1}{n.t}$dt=

= $\int_0^\infty e^{Lnt+Lnt^{1/n}-t^{1/n}} . \frac{1}{n.t}$dt= $\int_0^\infty e^{Ln(t.t^{\frac{1}{n}})-t^{1/n}} . \frac{1}{n.t}$dt=

$t^{1/n} = u$ t= u^n dt =nu $^{n-1}$ .du t → 0 u →0

t→ ∞ u→ ∞

= $\int_0^\infty e^{Ln(u^{n+1})-u} . \frac{1}{n.u^n}$.nu $^{n-1}$ .du= $\int_0^\infty e^{u.(Lnu-1)} . \frac{1}{u}$.du

n+1= u, dn = du, $\int_0^\infty e^{lnu!} . \left(\frac{1}{u}\right) . du$

x $^{n\,x}$ dx = dv → $\int_0^1 x^{n.x} = \int_0^1 dv$ →v=$\sum_{k=0}^{\infty} \frac{(-1)^k . n^k}{(k+1)^{k+1}}$

deduction that here

attention that the Stirling Approximation says : LnN ! ~N.lnN- N

therefore $\int_0^\infty e^{lnu!} . \left(\frac{1}{u}\right) . du$= $\int_0^\infty u! . \left(\frac{1}{u}\right) . du$=$\int_0^\infty (u-1)! . du$

On the other hand, attention :

$\frac{d}{dx}(x!) = \frac{d}{dx}(\Gamma(x+1))$=

= $\frac{d}{dx}\left(\int_0^\infty t^{x+1-1} . e^{-t} dt\right)$= $\int_0^\infty \frac{d}{dx} t^{x+1-1} . e^{-t} dt$=$\int_0^\infty t^x . lnt . e^{-t} dt$

Riemann integral

$$\chi = \int_0^a (u-1)!.du = \lim_{m\to a} \sum_{u=0}^{m} (u-1)!.\ \Delta\text{one}$$

$$\int_0^a .\Gamma(u).du = \int_0^a \left(\int_0^\infty e^{-r} r^{u-1}.dr\right) du =$$

$$= \int_0^\infty e^{-r} \left(\int_0^a r^{u-1} du\right).dr$$

If m →a ∃χ while if m→ ∞ ∄χ

Geometric series : $\sum_{u=0}^{m} a_u t^u$=S u where $a_0 = a_1 = \cdots = a_n$

S u =$a_0 + a_1.t + a_2.t^2 + a_3.t^3 + \cdots + a_m.t^m$

- tS u = -$a_{0.}t - a_1.t^2 - a_2.t^3 - a_3.t^4 - \cdots - a_m.t^{m+1}$

S u (1- t)= $a_0 - a_n.t^{n+1}$

a_p= $a_k.\ t^{p-k}$, $a_p = r.a_{p-1}$, if $a_0 = a_1 = \cdots = a_p$= 1

S u = $\frac{a_0 - a_0.t^u}{1-t}$= $\frac{1-1t^u}{1-t}$=$\frac{t^u-1}{t-1}$ convergent when $|t|$< 1

-1< t<1

Therefore :$\int_0^a r^{u-1} du = \lim_{m\to a} \sum_{u=0}^{m} r^{u-1}$. Δone

Where $a_u = du =$ Δu = ctnt= suppose 1

$\sum_{u=0}^{m} a_u.r^{u-1}$= $\sum_{u=0}^{m} a_u.\frac{r^u}{r}$ which using the geometric series gives S u =$\frac{1-1r^{u+1}}{r(1-r)}$

- r^{u+1} +1 | $-r^2 + r$

r^{u+1}-r^u $r^{u-1} + r^{u-2}$

r^u- r^{u-1} +1

…

So : = $\frac{1-1r^{u+1}}{r(1-r)}$($r^{u-1} + r^{u-2}$)+$\frac{-r^{u-1}+1}{-r^2+r}$

L=$\int_0^\infty e^{-r}\left[(r^{u-1} + r^{u-2}) + \frac{-r^{u-1}+1}{-r^2+r}\right].dr =$

$\frac{-r^{u-1}+1}{-r^2+r} = \frac{A}{r} + \frac{B}{1-r}$ A(1- r)+ Br=$-r^{u-1} + 1$

also **know that u=n+1,** therefore , if n= 1, **u= 2.**

A= 1 and r(B- A)= -r B= -1+1= 0

therefore $\int_0^\infty e^{-r}.r^{u-1}.dr + \int_0^\infty e^{-r}.r^{u-2}.dr$+$\int_0^\infty e^{-r}.\frac{1}{r}.dr$

$\int_0^\infty e^{-r}.r^{2-1}.dr + \int_0^\infty e^{-r}.r^{2-2}.dr$+$\int_0^\infty e^{-r}.\frac{1}{r}.dr$

L= $\Gamma(u)$+ $\Gamma(u-1)$+$\int_0^\infty e^{-r}.\frac{1}{r}.dr$

0

$\int e^{-r}.\frac{1}{r}.dr = \xrightarrow{\qquad\qquad}$ (1/r). ($-e^{-r}$)- $\int_0^\infty$($-e^{-r}$).-1/r^2 dr

j= 1/r dj= -1/r^2

di= $e^{-r}dr$ i= -e^{-r}

$\longrightarrow$ (1/r). ($-e^{-r}$)- (-1/r^2). e^{-r} + $\int_0^\infty$(e^{-r}).(2/r^3) dr

j$_2$= -1/r^2 dj$_2$= 2/r^3

di$_2$= $-e^{-r}dr$ i$_2$=e^{-r}

when going integrating by parts successively , we obtain a series next :

$(1/r).(-e^{-r}) - (-1/r^{2}).\, e^{-r} + (2/r^{3}).(-e^{-r}) + \dots + \int_0^{\infty} \quad dr$

Where I place “k” instead of “n” (to avoid repetition):

$\int e^{-r}.\frac{1}{r}.\, dr = \sum_{k=0}^{s}\frac{(-1)^{k}e^{-r}k!}{r^{k}}.1 \int \frac{(-1)^{k+1}e^{-r}(k+1)!}{r^{k+1}} + .dr$

Represented as : $\sum_{k=0}^{s}\frac{(-1)^{k}e^{-r}.k!}{r^{k}} + R_{k+1}(r)$

Let's remember that we conclude that $S_u$it was convergent only if$|r| < 1$

While from “k+1” to “ m" where m→ ∞

$R_{k+1}(r) = \int_{s=k+1}^{\infty}\frac{(-1)^{s}e^{-r}.(s)!}{r^{s}}$ dr= $\lim_{m\to\infty}\sum_{k+1}^{m}\frac{(-1)^{k+1}e^{-r}(k+1)!}{r^{k+1}} \to$

$\to (-1)^{\infty}e^{-r}(\infty)!.\, r^{-(\infty)} \to \infty.\, 0$ assuming that any number multiplied by 0 is 0.

$\sum_{k=1}^{s}\frac{(-1)^{s}e^{-r}.k!}{r^{k}}.\ \Delta$one

Therefore L= Γ(u)+ Γ(u+ 1)+ $\sum_{k=1}^{s}\frac{(-1)^{k}e^{-r}.k!}{r^{k}}$