Y=$\frac{e^{-ax}}{(1-e^{-ax})}$

We derive :

y= y

y'= (- a)(y+y^2)

and''=$(+a)^2.(y+3y^2+2y^3)$

and'''=$(-a)^3.(y+7.y^2+12.y^3+6.y^4)$

$y^{IV}=(+a)^4.(-y-y^2-14.y^2-14.y^3-36.y^3-36.y^4-$
$-24.y^4-24.y^5)$

…

Where suppose that n= nth derivative

When the Taylor Series is y= $\sum_{n=0}^{\infty}\frac{x^n.f^n(0)}{n!}$, we need to find an expression for the coefficients:

1 and 1 in y'

1 , 3 , 2 in y''

1 , 7 , 12, 6 in y'''

1 , 15, 50, 60, 24 in y^{IV}

…

Then we invent through trial and error:

y^1 y^2 y^3 y^4 y^5

n= 1: -1 -1

n= 2: + 1 + 3 +2

n= 3: -1 -7 -12 -6

n= 4: + 1 +15 +50 +60 + 24

that the series that meets $y^2$is: $a_n = (2.a_{n-1} + 1).(-1)^n$

while the one that complies $y^3$is: 2[(n-2).6+ $|n - 3|$].$(-1)^n$