Binomials with fractional exponent. (Newton's binomial).

Nº of coefficients ($\sum coef.$)		Exponent
1	1 term: 1 $(a+b)^0$	←0
(1'5)		0'5
1+1= 2	2 terms: 1a+1b $(a+b)^1$	←1
(3)		1'5
1+2+1=4	3 terms: $a^2+2ab+b^2$ $(a+b)^2$	←2
(6)		2'5
1+3+3+1=8	4 terms: $a^3+3a^2b+3ab^2+b^3$ $(a+b)^3$	←3
(12)		3'5
1+4+6+4+1=16	5 terms: $a^4+4a^3b+6a^2b^2+$ $+4ab^3+b^4$ $(a+b)^4$	←4

…

Nº of coefficients ($\sum coef.$)	$\sum exp.$	Exponent	
1	0	$(a+b)^0$	←0
(1'5)	1		0'5
1+1= 2	2	$(a+b)^1$	←1
(3)	4		1'5
1+2+1=4	6	$(a+b)^2$	←2
(6)	9		2'5
1+3+3+1=8	12	$(a+b)^3$	←3
(12)	16		3'5
1+4+6+4+1=16	20	$(a+b)^4$	←4

…

(a+b) ^{0} (a+b) $^{1/2}$ (a+b) ^{1} (a+b) $^{3/2}$ (a+b) ^{2} (a+b) $^{5/2}$ (a+b) ^{3} …

When the exponent is zero (n=0), that $\sum exponents$of each term of the binomial expansion is =0:$1^0 = 1$

However, when n=0'5, el $\sum exponents$= 1 and coefficient: 1'5:

In the following, we can try to make the average between nº terms "1" and nº terms "2" corresponding to (a+b)^0 and $(a+b)^1$ or also use the terms "1'5" and create a new expression:

Nº terms: and so on…

$(a+b)^{-1'5}$:	- x -x -x/2
(a+b) $^{-1}$:	-x -x
(a+b) $^{-0'5}$:	-x – x/2
(a+b) ^{0} :	- x
(a+b) $^{0'5}$:	x+ x/2
(a+b) ^{1} :	x +x
(a+b) $^{1'5}$:	x+ x+ /2
(a+b) ^{2} :	x+ x+ x
(a+b) $^{2'5}$:	x+ x+ x + x/2
(a+b) ^{3} :	x+ x+ x+ x+ x/2

And so on…

$(a+b)^{0'5}$:

1'5 →100 1'5 →100

1 → y y= 66'6 0'5 →z z= 33'3

Or: 1'5/2= ¾

3/4+3/4=1'5 for $\sum coef$.

$$(a+b)^{0.5} = \frac{[(1'5).1]+[(3/4)a^{\frac{1}{2}}+(3/4)b^{\frac{1}{2}}]}{2}$$

or $(a+b)^{0'5}$= 0'66. $a^{\frac{1}{2}} + 0'33.\ b^{\frac{1}{2}}$, which not seem to have enough sense because the first term has more importancy; and "a" and "b" have equal importance.

When n=1'5 el $\sum exponents$= 4:$\sum coef = 3$

$(3/2)a^2+(3/2)b^2$ and $(¾)a^{3/2}+(3/2)a^{1/2}b^{1/2}+(3/4)b^{3/2}$

or 3/2+1/2+1/2+3/2= 4

and if we add the two expressions and divide them by 2 we will obtain the expression of $(a+b)^{1'5}$:

$$(a+b)^{1'5} = \frac{\left[\left(\frac{3}{2}\right)a^2+\left(\frac{3}{2}\right)b^2\right]+\left[\left(\frac{3}{4}\right)a^{\frac{3}{2}}+\left(\frac{3}{2}\right)a^{\frac{1}{2}}b^{\frac{1}{2}}+\left(\frac{3}{4}\right).b^{\frac{3}{2}}\right]}{2}(?)$$

When n=2'5 the $\sum exponents$= 9 and$\sum coef = 6$

$(7/4)a^3+(5/2)a^{3/2}b^{3/2}+(7/4)b^3$

or 3+3/2+3/2+3= 9

$(1/2)a^{9/4}+(5/2)a^{5/4}b^1+(5/2)$to $^1 b^{5/4}+(1/2)b^{9/4}$

or 9/4+5/4+1+1+5/4+9/4= 9

(?)

When n=3'5 the $\sum exponents$= 16 and$\sum coef = 12$

(5/2) a^4 +(7/2) $a^{5/2}b^{3/2}$ +(7/2) $a^{3/2}b^{5/2}$ +(5/2)b^4

(7/4) $a^{16/5}$ +(5/2) $a^{7/5}b^{9/5}$ +(7/2) $a^{8/5}b^{8/5}$ +(5/2) $a^{9/5}b^{7/5}$ + +(7/4)$b^{16/5}$

If we also add them and divide by 2 we will obtain (a+b) $^{3'5}$

(?)

etcetera

Attention: some rules for the correct development of binomials:

1- Never n > than any of the exponents
2- The central terms of each polynomial representation contain the highest coefficients (understanding the coefficients as the numbers that precede each term; for example, when n=1'5, they would be 3/2 and 3/2 of the binomial and 3/4, 3/2 and ¾ of the binomial.
3- In the terms, no zero will ever be placed, neither in “a” nor in “b”.
4- Each exponent of each term “a” and “b” of each binomial must add up to the same.
5- also good to see that if we have an odd number of terms in the middle term, both “a” and “b” will have the same exponent. However, depending on the order in which the terms are, there will be an “inversion” of exponents.
6- The middle terms (“ab”) will have the largest coefficients; the opposite will happen with the exponents (as we can see in the previous examples).

And also a curiosity:

In the “key to” *wave functions* we see the functions that must be used or combined to obtain the expansion of the binomials $(a+b)^{1/2}$, $(a+b)^{3/2}$, $(a+b)^{5/2}$, $(a+b)^{7/2}$...

$\frac{N_1\varphi_1+N_2\varphi_2}{N_1+N_2}= \psi_{1,5}$coefficient

$\frac{N_2\varphi_2+N_3\varphi_3}{N_2+N_3}=\psi_3$

$\frac{N_3\varphi_3+N_4\varphi_4}{N_3+N_4}=\psi_6$

$\frac{N_4\varphi_4+N_5\varphi_5}{N_4+N_5}=\psi_{12}$

...

or if necessary, taking the Taylor polynomial where $P(x)= (1+ x)^{1/2}$

and $P_n(x)= P(a)+ \frac{(x-a)}{1!}P'(a) + + \frac{(x-a)^2}{2!}P''(a)+ \frac{(x-a)^3}{3!}P'''(a)+...+\frac{(x-a)^n}{n!}P^n(a)$

$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k} a^{nk} . b^k$

$\binom{n}{m}= \frac{n!}{m!.(n-m)!}$where $n \geq m$.