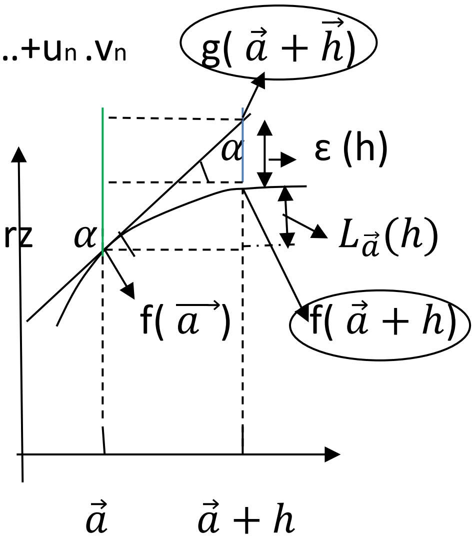
PARTIAL DERIVATIVES IN 3-D:

Knowing that $<\underset{u}{\rightarrow}| \ \underset{v}{\rightarrow}>= u_1 .v_1 + u_2 .v_2 + \ldots + u_n .v_n$



And since according to Cauchy -Schwarz

$|\vec{x}.\vec{y}| = \langle\vec{x}|\vec{y}\rangle \leq \|\vec{x}\|.\|\vec{y}\|,$

$<D_n f\left(\underset{a}{\rightarrow}\right)| \ \underset{h}{\rightarrow}> \leq \left\|D_n f\left(\underset{a}{\rightarrow}\right)\right\|.\|\vec{h}\|$

Where n=2

$\langle\vec{x}|\vec{y}\rangle = |\sum_{i=1}^{n} x_i.y_i|$

If f is differentiable, $\vec{h} = h.\hat{u}$ and $L_{\vec{a}}(\hat{u})$ it is the same as $Df_{\vec{a}}(\hat{u})$

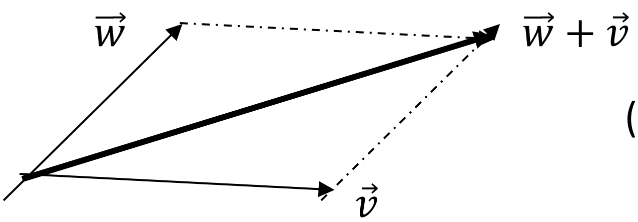
and $L_{\vec{a}}(h) = <D_n f\left(\underset{a}{\rightarrow}\right)| \ \underset{h}{\rightarrow}>$and $D_n f\left(\underset{a}{\rightarrow}\right)$are the slopes. we define $\vec{\nabla}f(\vec{a}) = (D_1 f\left(\underset{a}{\rightarrow}\right), D_2 f\left(\underset{a}{\rightarrow}\right))$ and $\vec{h} = (h_1, h_2)$

when we treat $\hat{u} = \hat{e}_n$

$L_{\vec{a}}(\vec{h})$=$L_{\vec{a}}(\sum_{n=1}^{2} h_n\hat{e}_n) = \sum_{n=1}^{2} h_n.L_{\vec{a}}(\hat{e}_n)$

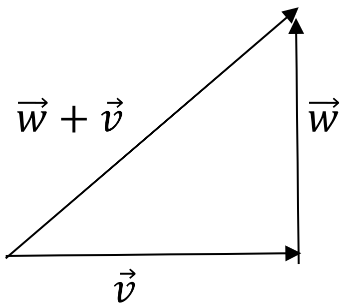
$Df_{\vec{a}}(\hat{e}_n) = L_{\vec{a}}(\hat{e}_n) = D_n f\left(\underset{a}{\rightarrow}\right)$

know that the sum of vectors gives a vector, and it is :

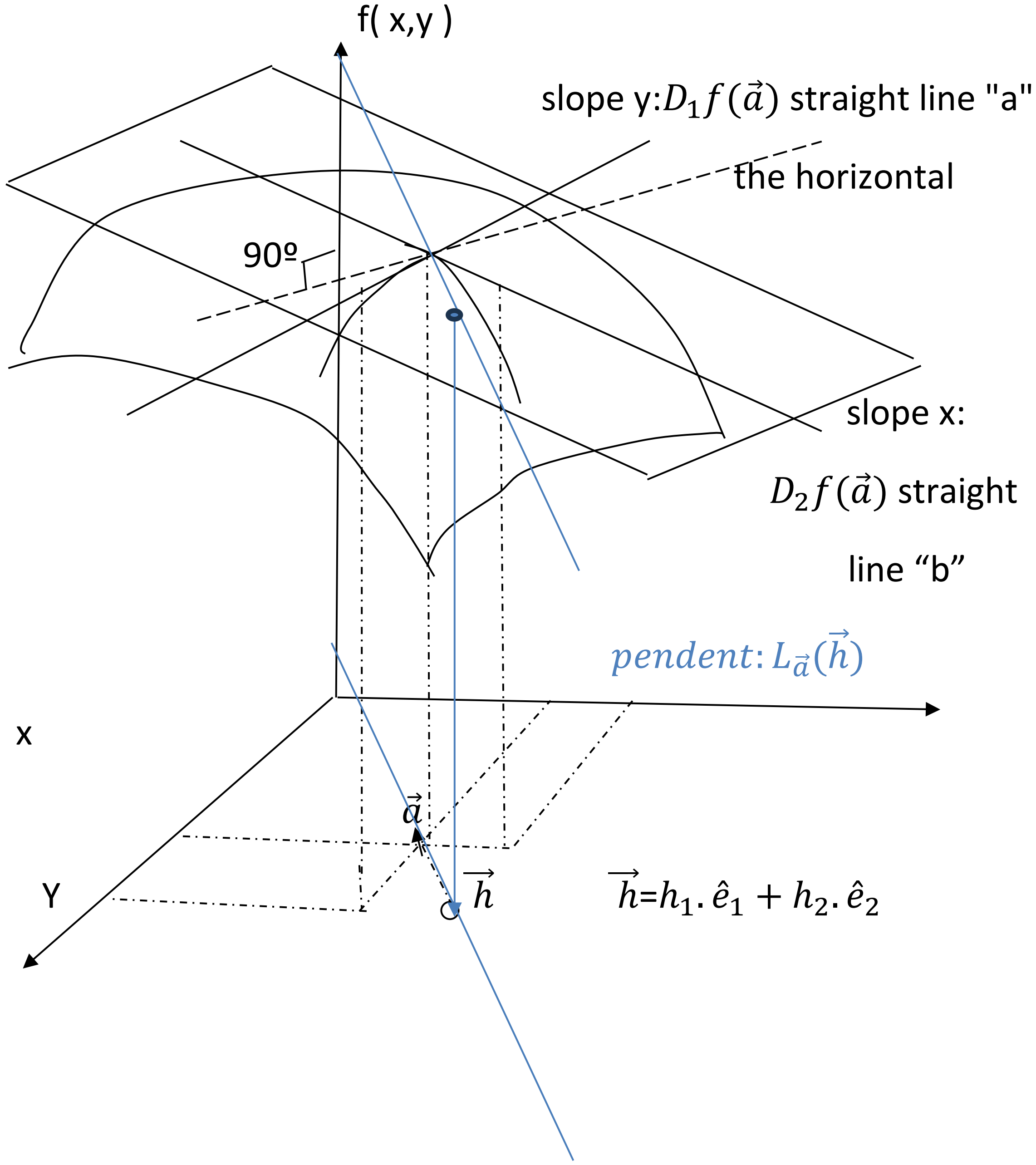


(ع)

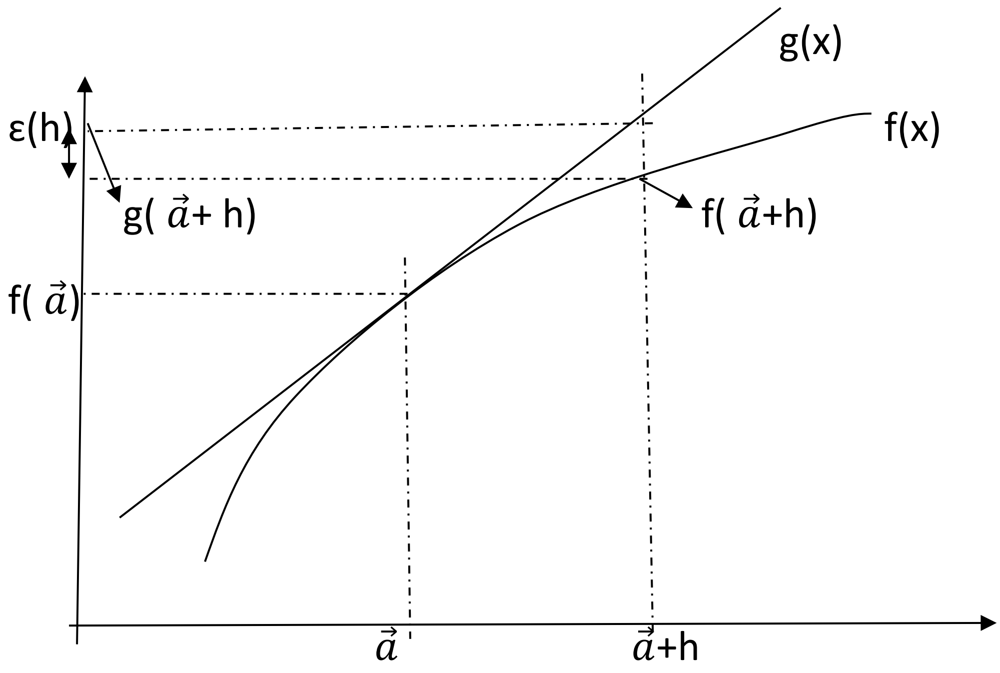
while if we work with distances (and we are in front of triangles rectangles):



$$\|\vec{w} + \vec{v}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2$$



f(x,y)
slope y:$D_1 f(\vec{a})$ straight line "a"
the horizontal
90º
slope x:
$D_2 f(\vec{a})$ straight
line “b”
$pendent: L_{\vec{a}}(\vec{h})$
x
Y
$\vec{a}$
$\vec{h}$
$\vec{h}=h_1.\hat{e}_1 + h_2.\hat{e}_2$



g($\vec{a}$+h)- f($\vec{a}$)= f'($\vec{a}$).h ε (h)= g($\vec{a}$+h)- f($\vec{a}$+h)

$\lim_{h\to 0}\left|\frac{\varepsilon(\mathrm{h})}{h}\right|$= 0= $\lim_{h\to 0}\left|\frac{\mathrm{g}(\vec{a}+\mathrm{h})-\mathrm{f}(\vec{a}+\mathrm{h})}{h}\right|$=$\lim_{h\to 0}\frac{f(\vec{a})+\mathrm{f}'(\vec{a}).\mathrm{h}-\mathrm{f}(\vec{a}+\mathrm{h})}{h}$= 0

with the signs changed : - $\mathrm{f}'(\vec{a})$=$\lim_{h\to 0}\frac{f(\vec{a})-\mathrm{f}(\vec{a}+\mathrm{h})}{h}$

$L_{\vec{a}}\left(\vec{h}\right) = L_{\vec{a}}(h.\hat{u}) = h.L_{\vec{a}}(\hat{u})$when$\lim_{\vec{h}\to 0}\frac{\left|f\left(\vec{a}+\vec{h}\right)-f(\vec{a})-L_{\vec{a}}\left(\vec{h}\right)\right|}{\|\vec{h}\|}$

and, as we have seen , $\vec{h}$= $h.\hat{u}$

$L_{\vec{a}}(\vec{h})$=$L_{\vec{a}}(\sum_{n=1}^{2} h_n\hat{e}_n) = \sum_{n=1}^{2} h_n.L_{\vec{a}}(\hat{e}_n)$

$Df_{\vec{a}}(\hat{e}_n) = L_{\vec{a}}(\hat{e}_n) = D_n f\left(\underset{a}{\to}\right)$=slope, and the sum of the two perpendicular lines “a” and “b” gives a resultant ,

according to (ƺ).