

Function $f(x) = \frac{1}{1+x^2}$

	<u>x=0</u>	
$f(x) = \frac{1}{1+x^2}$	1	a_0
$f'(x) = \frac{-2x}{(1+x^2)^2}$	0	a_1
$f''(x) = \frac{-2 \cdot (1+x^2)^2}{(1+x^2)^4} - \frac{-2x \cdot 2 \cdot (1+x^2) \cdot 2x}{(1+x^2)^4}$	-2	a_2
$f'''(x) = 0 - \frac{-2 \cdot 2 \cdot (1+x^2) \cdot 2x}{(1+x^2)^4}$	0	a_3
$f^{IV}(x) = \frac{8 \cdot (1+x^2)^3}{(1+x^2)^6} - \frac{8 \cdot 3 \cdot (1+x^2)^2 \cdot 2x}{(1+x^2)^6}$	8	a_4
$f^V(x) = \frac{-8 \cdot 3 \cdot (1+x^2)^2 \cdot 2x}{(1+x^2)^6}$	0	a_5
$f^{VI}(x) = \frac{-48 \cdot (1+x^2)^3}{(1+x^2)^8} - 0$	-48	a_6
$f^{VII}(x) = \frac{+48 \cdot 4 \cdot (1+x^2)^3 \cdot 2x}{(1+x^2)^8}$	0	a_7
$f^{VIII}(x) = \dots$	384	a_8
$f^{IX}(x) = \dots$	0	a_9
$f^X(x) = \dots$	-3840	a_{10}

...

According to Taylor: $f(x) = \sum_{n=0}^{\infty} \frac{x^n \cdot f^n(0)}{n!}$

So, following the thread that only ones a_{2n} (or pairs) have value, we devised the following geometric series:

$$a_{2n} = a_{2n-2} \cdot (-2 \cdot n)$$

We know that the sum in geometric series is: $S_n = \frac{a_0 - a_0 \cdot r^{n+1}}{1-r}$

Therefore: $a_{2n} = a_{2n-2} \cdot r^{2n-(2n-2)}$

Then we start the Taylor Series for $n=1$ and add previously a_0 .

$$a_2 = a_0 \cdot r^2, a_4 = a_2 r^2 = a_0 \cdot r^4, a_6 = a_0 \cdot r^6 \dots$$

We can also assimilate $\sum_{n=1}^{\infty}$ to S_{2n}

$$\begin{aligned} S_{2n} &= a_0 r^2 + a_0 r^4 + \dots + a_0 r^{2n} \\ + (-r^2) \cdot S_{2n} &= a_0 r^4 + a_0 r^6 + a_0 r^8 + \dots + a_0 r^{2n+2} \\ \hline (1 - r^2) \cdot S_{2n} &= a_0 \cdot r^2 - a_0 \cdot r^{2n+2} \end{aligned}$$

And we will obtain: $S_{2n} = \frac{a_0 \cdot r^2 - a_0 \cdot r^{2n+2}}{1 - r^2},$

which means that $(-2n) = r^2$ and $r^{2n} = (-2n)^n$

$S_{2n} = \frac{(-2n) - [(-2n) \cdot (-2n)^n]}{1 - (-2n)}$ which, expanded, turns out to be:

$$S'_{2n} = \frac{x^{2n}}{n!} \frac{(-2n) - [(-2n) \cdot (-2n)^n]}{1 - (-2n)}$$

$$f(x) = \frac{1}{1+x^2} = 1 + S'_{2n}$$