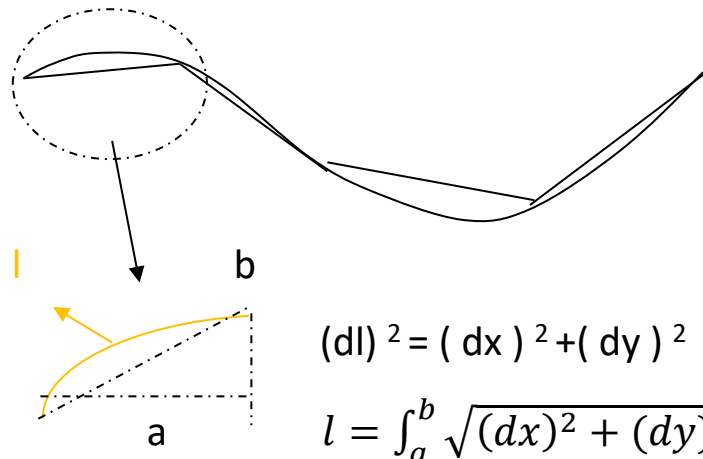


ARCLENGTH:



and knowing the definition of derivative: " $f'(x) = dy / dx$ "

$$l = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

In a non-rectangular triangle $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$

$$x(t) = u(t) \text{ and } y(t) = v(t) \quad L = \int_a^b \sqrt{\left\| \frac{d}{dt} \vec{r}(u(t), v(t)) \right\|^2} dt =$$

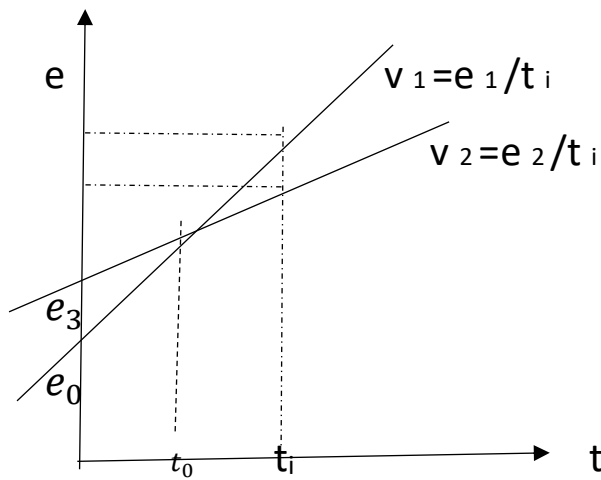
$$= \int_a^b \sqrt{u'(t)^2 \vec{r}_u \vec{r}_u + 2u'(t)v'(t) \vec{r}_u \vec{r}_v + v'(t)^2 \vec{r}_v \vec{r}_v} dt$$

Where $\vec{r}_u$ and $\vec{r}_v$ are the respective unit vectors of $u(t)$ and $v(t)$.

$$dL^2 = (du dv) \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} du \\ dv \end{bmatrix} \text{ where } E = \vec{r}_u \vec{r}_u, F = \vec{r}_u \vec{r}_v, G = \vec{r}_v \vec{r}_v$$

and where $\vec{r} = \vec{u} + \vec{v}$ if $\vec{u}$ were perpendicular to $\vec{v}$ the term

$$2u'(t)v'(t)\vec{r}_u \vec{r}_v \text{ would not exist}$$

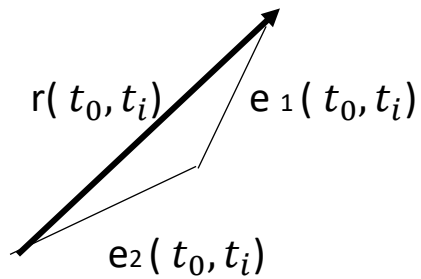


$$e_1 = v_1 \cdot t + e_0 \quad \text{where} \quad e_0 + v_1 \cdot t_0 = v_2 \cdot t_0 + e_3$$

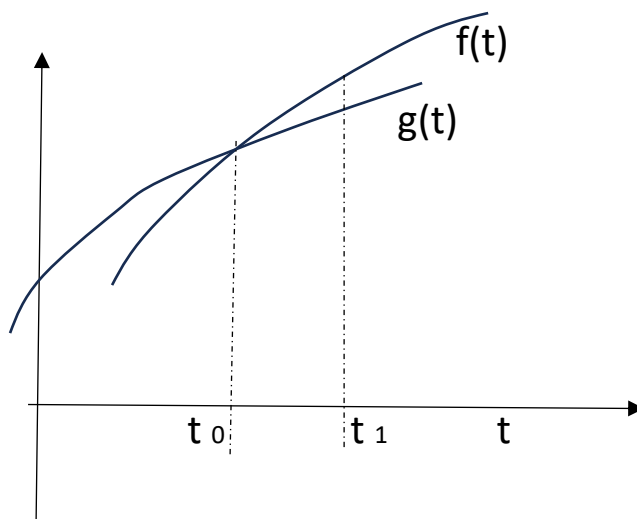
$v_1 = \text{slope of } e_1$

$$e_2 = v_2 \cdot t + e_3 \quad v_2 = \text{slope of } e_2$$

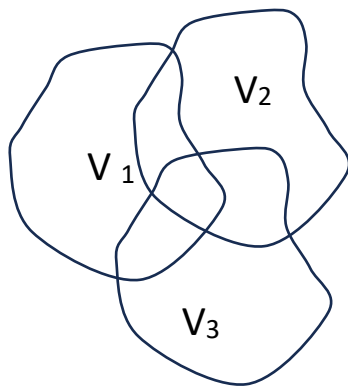
when $e_1(t_0, t_i)$ and $e_2(t_0, t_i)$ are added, we obtain $r(t_0, t_i)$:



and in e_1 and e_2 curves, the same thing happens :



SET THEORY:



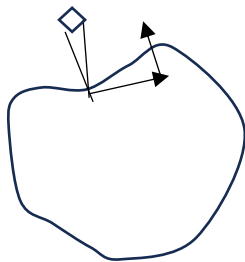
$$V_1 \cup V_2 \cup V_3 - V_1 \cap V_2 - V_1 \cap V_3 - V_2 \cap V_3 - V_1 \cap V_2 \cap V_3$$

Once you know how to calculate length, now it's time for surface area and volume :

$$V_m = \int_{\vec{m}} \sqrt{|\delta_{i,j}|} . dx_0 . dx_1 . dx_2 . dx_3 \quad i,j = 0,1,2,3$$

$$\vec{m} = \vec{dx}_0 + \vec{dx}_1 + \vec{dx}_2 + \vec{dx}_3 \quad \text{vectors unitary : } \widehat{u}_0, \widehat{u}_1, \widehat{u}_2, \widehat{u}_3$$

AREA



and we will find the body area of

3-D. If we multiply it by one more term "h" , we will find the volume :

let's remember **arclength** when the triangle is not a rectangle :

$u(t)$ and $v(t)$

$$L = \int_a^b \sqrt{u'(t)^2 \vec{r}_u \vec{r}_u + 2u'(t)v'(t) \vec{r}_u \vec{r}_v + v'(t)^2 \vec{r}_v \vec{r}_v} dt$$

On **surfaces** :

we will call this one 3x3 matrix "M". where

$(a \ b \ c) = (dx_0 \ dx_1 \ dx_2)$ where $\vec{n} = \overrightarrow{dx_0} + \overrightarrow{dx_1} + \overrightarrow{dx_2}$

and the vectors units of each $\overrightarrow{dx_n}$ will be: $\widehat{u_0}, \widehat{u_1}, \widehat{u_2}$

$$dS^3 = (dx_0^{3/2} \quad dx_1^{3/2} \quad dx_2^{3/2}) \cdot "M" \cdot \begin{pmatrix} dx_0^{3/2} \\ dx_1^{3/2} \\ dx_2^{3/2} \end{pmatrix} \quad (i)$$

the "M": $A_{000} = \widehat{u_0^3}$, $B_{001} = B_{100} = \widehat{u_0^2} \widehat{u_1}$, $C_{002} = C_{200} = \widehat{u_0^2} \widehat{u_2}$

$D_{101} = \widehat{u_0} \widehat{u_1^2}$, $E_{102} = E_{201} = \widehat{u_0} \widehat{u_1} \widehat{u_2}$, $F_{202} = \widehat{u_0} \widehat{u_2^2}$

Where $\widehat{u_0}, \widehat{u_1}, \widehat{u_2}$ do not form right angles with each other.

Let's assume that according to arclength : $u(t) = x_0(t)$, $v(t) = x_1(t)$

While for Surfaces we add another variable: $x_2(t, \varphi)$

Let's assume an example : $x_0(t) = \alpha t$

$$x_1(t) = \beta t$$

$$x_2(t, \varphi) = \gamma t \varphi$$

be carefull because 3 lines are enough to obtain a surface

$$M: \begin{bmatrix} A_{000} & B_{001} & C_{002} \\ B_{100} & D_{101} & E_{102} \\ C_{200} & E_{201} & F_{202} \end{bmatrix}$$

Now, operating algebraically (i), we will obtain :

$$dS^3 = dx_0^3 \hat{u}_0^3 + 2dx_0^{3/2} dx_1^{3/2} \hat{u}_0^2 \hat{u}_1 + 2dx_2^{3/2} dx_0^{3/2} \hat{u}_0^2 \hat{u}_2 + 2dx_1^{3/2} dx_2^{3/2} \hat{u}_0 \hat{u}_1 \hat{u}_2 + dx_1^3 \hat{u}_0 \hat{u}_1^2 + dx_2^3 \hat{u}_0 \hat{u}_2^2$$

$$\text{if now } dS = \sqrt[3]{(\dots)} \cdot \frac{dt}{dt} \rightarrow \sqrt[3]{\frac{(\dots)}{dt^3}} \cdot dt \rightarrow$$

$$\sqrt[3]{\frac{dx_0^3 \hat{u}_0^3}{dt^3} + 2 \frac{dx_0^{3/2}}{dt^{3/2}} \frac{dx_1^{3/2}}{dt^{3/2}} \hat{u}_0^2 \hat{u}_1 + 2 \frac{dx_2^{3/2}}{dt^{3/2}} \frac{dx_0^{3/2}}{dt^{3/2}} \hat{u}_0^2 \hat{u}_2 +$$

$$+ 2 \cdot \frac{dx_1^{3/2}}{dt^{3/2}} \frac{dx_2^{3/2}}{dt^{3/2}} \hat{u}_0 \hat{u}_1 \hat{u}_2 + \frac{dx_1^3}{dt^3} \hat{u}_0 \hat{u}_1^2 + \frac{dx_2^3}{dt^3} \hat{u}_0 \hat{u}_2^2}$$

In each term let's follow the next one mechanism :

$$\text{Example: } \frac{[d(\alpha.t)]^3}{dt^3} = \left(\frac{d(\alpha.t)}{dt} \right)^3 = \alpha^3, \quad 2 \cdot \left(\frac{d(\alpha.t)}{dt} \right)^{3/2} \cdot \left(\frac{d(\beta.t)}{dt} \right)^{3/2} = \alpha^{3/2} \beta^{3/2} \text{ and so on until the end.}$$

$$\alpha^3 \hat{u}_0^3 + 2\alpha^{3/2} \beta^{3/2} \hat{u}_0^2 \hat{u}_1 + 2\alpha^{3/2} (\gamma\varphi)^{3/2} \hat{u}_0^2 \hat{u}_2 + 2\beta^{3/2} (\gamma\varphi)^{3/2} \hat{u}_0 \hat{u}_1 \hat{u}_2 + \beta^3 \hat{u}_0 \hat{u}_1^2 + (\gamma\varphi)^3 \hat{u}_1 \hat{u}_2^2$$

We put limits on "t" and "φ": $t_1 = 1, t_2 = 2, \varphi_1 = 1, \varphi_2 = 3$

$$S = \int_{\varphi_1}^{\varphi_2} \left(\int_{t_1}^{t_2} \sqrt[3]{(\dots)} dt \right) d\varphi \rightarrow S = \int_{\varphi_1}^{\varphi_2} \left(\left[\sqrt[3]{(\dots)} \cdot t \right]_{t_1}^{t_2} \right) d\varphi \rightarrow$$

$$\rightarrow S = \int_{\varphi_1}^{\varphi_2} \left(\sqrt[3]{(\dots)} \right) d\varphi \rightarrow S = \int_{\varphi_1}^{\varphi_2} \left(\sqrt[3]{(a \cdot \varphi^3 + b \cdot \varphi^{3/2} + c)} \right) d\varphi$$

$$a=\gamma^3, b=2.(\alpha^{3/2}\gamma^{3/2} + \beta^{3/2}\gamma^{3/2}),$$

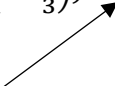
$$c=\alpha^3 + 2\alpha^{3/2}\beta^{3/2} + \beta^3=(\alpha^{3/2} + \beta^{3/2})^2$$

$$(\eta.\varphi^{3/2}+\varepsilon)^{2/3} = \text{Newton binomial} =$$

$$= \binom{2/3}{0} \cdot (\eta.\varphi^{\frac{3}{2}})^{2/3} \varepsilon^0 + \binom{2/3}{1/3} \cdot (\eta.\varphi^{\frac{3}{2}})^{1/3} \varepsilon^{1/3} + \binom{2/3}{2/3} \cdot (\eta.\varphi^{\frac{3}{2}})^0 \varepsilon^{2/3}$$

$$\binom{2/3}{0} = \frac{(\frac{2}{3})!}{(\frac{2}{3}-0)!.0!} = \frac{(\frac{2}{3})!}{(\frac{2}{3}-\frac{2}{3})!!(\frac{2}{3})!} \binom{2/3}{2/3} = 1$$

While

$$\binom{2/3}{1/3} = \frac{(\frac{2}{3})!}{(\frac{1}{3})!.(\frac{1}{3})!} = \frac{\Gamma(\frac{2}{3}+1)}{(\Gamma(1+\frac{1}{3}))(\Gamma(1+\frac{1}{3}))} =$$


$$\Gamma(x+1)=x!=x.\Gamma(x)$$

$$\Gamma(n+1/3)=\Gamma(1/3).\frac{(3n-2)!!!}{3^n}$$

$$\Gamma(1/3)=\Gamma(1-2/3)$$

$$\Gamma(1-2/3).\Gamma(2/3)=\frac{\pi}{\sin[\pi(\frac{2}{3})]}.$$

$$\frac{(\frac{2}{3}).\Gamma(\frac{2}{3}).\sin^2(\frac{2\pi}{3}).(\Gamma(\frac{2}{3}))^2}{(1/9).\pi^2} = \frac{6}{\pi^2} \cdot \sin^2\left(\frac{2\pi}{3}\right) \cdot (\Gamma\left(\frac{2}{3}\right))^3 = "C"$$

$$(a \cdot \varphi^3 + b \cdot \varphi^{\frac{3}{2}} + c) = (\eta \cdot \varphi^{3/2} + \varepsilon)^2$$

$$a = \eta^2, b = 2 \cdot (\eta) \cdot \varepsilon, c = \varepsilon^2$$

$$S = \int_{\varphi_1}^{\varphi_2} \left(\sqrt[3]{(\eta \cdot \varphi^{3/2} + \varepsilon)^2} \right) d\varphi$$

$$(\eta \cdot \varphi^{3/2} + \varepsilon)^{2/3} = 1 \cdot \eta^{2/3} \cdot \varphi + "C" \cdot \varepsilon^{1/3} \eta^{1/3} \varphi^{1/2} + \varepsilon^{2/3}$$

$$(\eta \cdot \varphi^{3/2} + \varepsilon)^2 = \eta^2 \varphi + 2 \cdot \eta \cdot \varepsilon \cdot \varphi^{3/2} + \varepsilon^2$$

$$S = \int_{\varphi_1}^{\varphi_2} (\eta \cdot \varphi^{3/2} + \varepsilon)^{2/3} d\varphi =$$

$$= \int_{\varphi_1}^{\varphi_2} (\eta^{2/3} \cdot \varphi + "C" \cdot \varepsilon^{1/3} \eta^{1/3} \varphi^{1/2} + \varepsilon^{2/3}) d\varphi =$$

$$\sqrt[3]{a} = \eta \rightarrow \eta^{2/3} = \sqrt[3]{a}$$

$$\sqrt[3]{b} = (2^{1/3} \eta^{1/3} \varepsilon^{1/3}) \text{ by chance } 2^{1/3} = "C" ?$$

$$\varepsilon^{2/3} = \sqrt[3]{c}$$

$$S = \int_{\varphi_1}^{\varphi_2} (\sqrt[3]{a} \cdot \varphi + \sqrt[3]{b} \cdot \varphi^{3/2} + \sqrt[3]{c}) d\varphi =$$

$$= \left[\frac{\sqrt[3]{a}}{2} \varphi^2 + \frac{\sqrt[3]{b}}{3/2} \varphi^{3/2} + \sqrt[3]{c} \varphi \right]_1^3 = \left[\frac{\sqrt[3]{a}}{2} \cdot 9 + \frac{2\sqrt[3]{b}}{3} \cdot 3 \cdot \sqrt{3} + 3 \cdot \sqrt[3]{c} \right] -$$

$$- \left[\frac{\sqrt[3]{a}}{2} + \frac{2\sqrt[3]{b}}{3} + \sqrt[3]{c} \right]$$

Knowing that:

$$a = \gamma^3, b = 2 \cdot (\alpha^{3/2} \gamma^{3/2} + \beta^{3/2} \gamma^{3/2}),$$

$$c = \alpha^3 + 2\alpha^{3/2} \beta^{3/2} + \beta^3 = (\alpha^{3/2} + \beta^{3/2})^2$$

we can end up replacing "a, b, c"