ATTENTION: FACTORIAL AND PRODUCTION "WORKERS":

$$\frac{(n+1)!}{n!} \sim \prod_{k=1}^{n} \frac{(k+1)}{k}.\left(\frac{n+1}{n+1}\right) \rightarrow \prod_{k=1}^{n} \frac{(k+1)}{k}.\left(\frac{n}{n+1} + \frac{1}{n+1}\right)$$

$$\mathbf{n} \equiv k \xrightarrow{\quad} \frac{(k+1)!}{(k+1)}.\frac{(k+1)}{k!} \rightarrow k!.\frac{(k+1)}{k!} = (k+1)$$

$$\rightarrow \frac{(k+1)!.n}{k!.(n+1)} + \frac{(k+1)!}{k!.(n+1)} \quad \rightarrow \frac{(k+1)}{(k)}\frac{(k+1)}{(k+1)}\prod_{k=1}^{n-1}\frac{(k+1)}{k} \rightarrow$$

$$\rightarrow \frac{(k+1)}{k}\frac{k!}{(k-1)!}$$

Factorial:

(n+k)!= (n+k).((n- 1)+ k).((n- 2)+ k).((n- 3)+k)… k.(k-1).(k-2).. 1

$$\frac{(n+k)!}{(n+k).((n-1)+k).((n-2)+k)...(k+1).k} = (k-1)! = \Gamma(k)$$

$$\lim_{n\to\infty} \frac{(n+k)!}{(n+k).\big((n-1)+k\big)\ldots(1+k).k.(k-1)!\ldots 1} = 1$$

Derivative:

$$\frac{\partial}{\partial x^3}(x^{10}) = 10.9.8.\,x^7 = \frac{10!}{(10-3)!}.\,x^7 = \frac{n!}{(n-\alpha)!}.\,x^{n-\alpha} = \frac{\Gamma(n+1)}{\Gamma(n-\alpha+1)}.\,x^{n-\alpha}$$

Thus : $\frac{\partial^{1/2}}{\partial x^{1/2}}(x^0) = \frac{\Gamma(1)}{\Gamma(1/2)}.\,x^{-1/2}$

Taylor series :

From the integral:

I= $\int \frac{1}{(x-1)} + \frac{1}{x(x-1)} + \frac{1}{x^2(x-1)} + \frac{1}{x^3(x-1)} + \frac{1}{x^4(x-1)}$ ...=

I= $\int \frac{(x-2)!}{(x-1)!} + \frac{(x-2)!}{x!} + \frac{(x-2)!}{x.x!} + \frac{(x-2)!}{x^2.x!} + \frac{(x-2)!}{x^3.x!}$ ...=

I=$\int \frac{(x-2)!}{(x-1)!} + \frac{(x-2)!}{x!}[1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \cdots]$

According to Taylor: $\sum_{n=0}^{N}(\frac{1}{x})^n$=$\sum_{n=0}^{\infty} a_n.r^{n-1}$

since$(\frac{1}{x})^n = (\frac{1}{x})^{n-1}.(\frac{1}{x})^1$

Therefore : r^{n-1}= $(\frac{1}{x})^{n-1}$and a_n=$(\frac{1}{x})^1$

Yes$|r| < 1 \quad convergeix$

si $|r|$ ≥1 diverges

$a_1 = a_0.r^{1-0} \qquad a_2 = a_0.r^2 ... a_0 = a_n$

$S_n = \frac{a_0}{r} + a_1 + a_2.r^1 + a_3.r^2 + \cdots + a_n r^{n-1}$

$S_n(-r) = -a_0 - a_1.r^1 - a_2.r^2 - \cdots - a_n r^n$

+

S_n(1- r)= $\frac{a_0}{r}$-$a_n r^n \;\; \rightarrow \;\; S_n$=$\frac{a_0.(\frac{1}{r} - r^n)}{(1-r)}$

$$\boxed{\int \frac{1}{x^n} dx = \int x^{-n}\, dx = \frac{1}{(n+1)} x^{-n+1}}$$

Now we assimilate the integral to the series geometric :

add up to the term enèssim $=S_n = \frac{a_1.(1-r^n)}{(1-r)}$

$$S_n = \frac{\frac{1}{x}.\left(\frac{1}{(\frac{1}{x})} - \frac{1}{x^n}\right)}{1-\left(\frac{1}{x}\right)} = \frac{1}{x}.\frac{\left(x-\frac{1}{x^n}\right)}{\frac{(x-1)}{x}} = \frac{\left(\frac{x^{n+1}-1}{x^n}\right)}{(x-1)} = \left(\frac{(x^{n+1}-1)}{(x^n)(x-1)}\right) =$$

$$= \frac{x^{n+1}}{(x^{n+1}-x^n)} - \frac{1}{(x^{n+1}-x^n)}$$

Note that a binomial is not a Taylor sum:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k}. a^n. b^{n-k} \neq \sum_{n=0}^{N} a_n. x^n = f(x)$$

where $a_n = \frac{f^n(0)}{n!}$