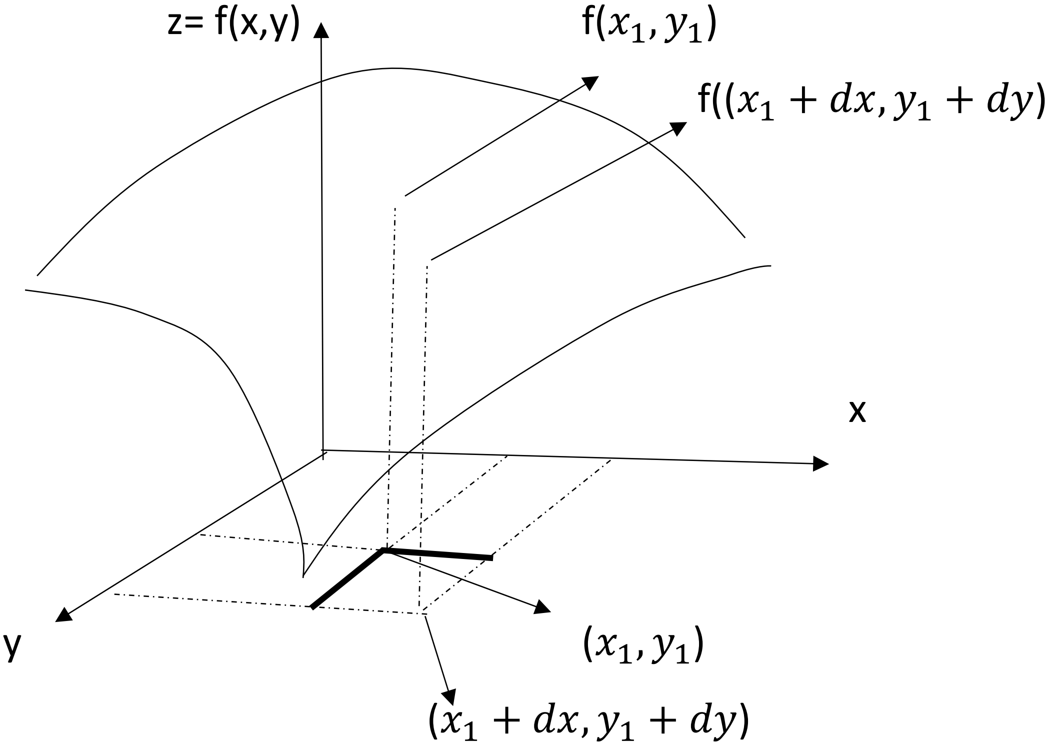
Taylor Series in partial derivatives:

in 2 variables:



According to Taylor: $\sum_{n=0}^{\infty} \frac{x^n . f^n(a)}{n!}$

And the defintion of derivative:

F'(x_1,y_1)= $\lim_{\Delta p \to "n"} \frac{f(x+dx, y+dy) - f(x,y)}{\Delta p}$ on Δp=$\sqrt{dx_1^2 + dy_1^2}$

Depending of the value of "n" the slope will reach to zero or ∞.

l'order of the derivative

$$f^n(x_1, y_1).\,\Delta p$$

$f^n(x_1, y_1).\,\Delta p = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{1.\partial^2 f}{2!.\partial x^2} dx^2 \frac{2.\partial^2 f}{2!.\partial x \partial y} dxdy +$
$+ \frac{1.\partial^2 f}{2!.\partial y^2} dy^2 + + \frac{1.\partial^3 f}{3!.\partial x^3} dx^3 + \frac{3.2.\partial^3 f}{3!.\partial x^2 \partial y} dx^2 dy^1 +$
$+ \frac{3.2.\partial^3 f}{3!.\partial x \partial y^2} dx^1 dy^2 + \frac{1.\partial^3 f}{3!.\partial y^3} dy^3 + \ldots$

Which gives: $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \to 2.\frac{\partial^2 f}{\partial x \partial y}$

(THEOREM SCHWARZ)

And the same: $\frac{\partial}{\partial x}\left(2.\frac{\partial^2 f}{\partial x \partial y}\right) \to 2.\frac{\partial^3 f}{\partial x \partial x \partial y} = 2.\frac{\partial^3 f}{\partial x \partial y \partial x} = 2.\frac{\partial^3 f}{\partial y \partial x \partial x} \to$

$\to 3.2.\frac{\partial^3 f}{3!.\partial x^2 \partial y}$.

And with $\frac{3.2.\partial^3 f}{3!.\partial x \partial y^2}$ produces the same mecanism.

Watching out:

$x^n \equiv dx^n$, $dx^{n-1} dy^1$, $dx^{n-2} dy^2$,

Till $dx^1 dy^{n-1}$ and dy^n