

TRIANGULAR I INVERSE INEQUALITY:

$$|x + y| \leq |x| + |y|$$

$$|x - y| \leq |x| + |y| \quad |x| \leq a \rightarrow -a \leq x \leq a$$

$$d(P, Q) = \sqrt{|a_1 - b_1|^2 + |a_2 - b_2|^2}$$

$$|d(P, Q)|^2 = |a_1 - b_1|^2 + |a_2 - b_2|^2$$

Començem per $(|a - b|)^2 \leq (|a| + |b|)^2 = |a|^2 + 2 \cdot |a||b| + |b|^2$
si $a = -2$ i $b = 4$: $|-2 - 4| \leq |-2| + |4|$: $|-6| \leq 2 + 4$ Ok!

INEQUALITY CAUCHY-SCHWARZ:

$$(a_1 - b_1)^2 + (a_2 - b_2)^2 \leq$$

$$\geq (a_1^2 + a_2^2) + 2 \cdot \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2} + b_1^2 + b_2^2 \rightarrow$$

$$\rightarrow (a_1^2 - 2a_1b_1 + b_1^2) + (a_2^2 - 2a_2b_2 + b_2^2) \leq (a_1^2 + a_2^2) +$$

$$2 \cdot \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2} + b_1^2 + b_2^2 \rightarrow$$

$$\rightarrow -2a_1b_1 - 2a_2b_2 \leq 2 \cdot \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}, \text{ which is the}$$

inequality Cauchy- Schwarz, or otherwise:

$$|\sum_{i=1}^n a_i b_i| \leq \sqrt{\sum_{i=1}^n a_i^2 \cdot \sum_{i=1}^n b_i^2}$$