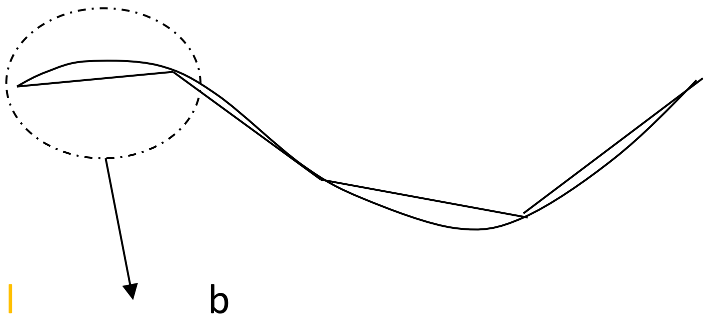
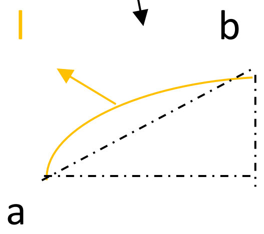
ARCLENGTH:





$(dl)^2 = (dx)^2 + (dy)^2$

$$l = \int_a^b \sqrt{(dx)^2 + (dy)^2}$$

i sabent la dedinició de derivada: “f’(x)= dy/dx”

$$l = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

En triangle no rectangle $L = \int_a^b \sqrt{\left(x'(t)\right)^2 + (y'(t))^2 dt}$

x(t)= u(t) i y(t)=v(t) $L = \int_a^b \sqrt{\left\| \frac{d}{dt}\vec{r}(u(t), v(t)) \right\|}\,dt =$

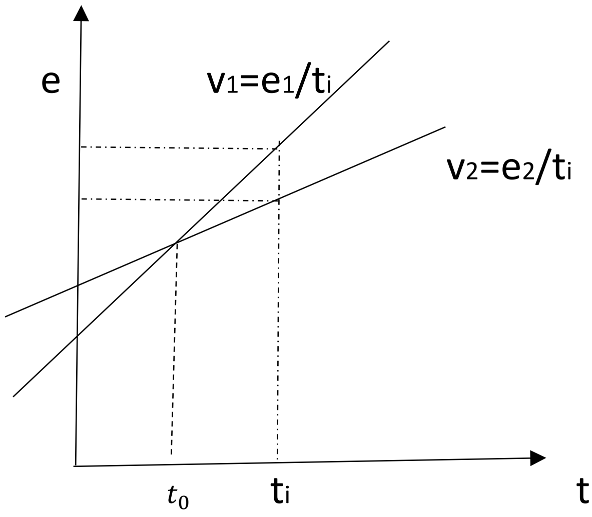
$$= \int_a^b \sqrt{u'(t)^2\overrightarrow{r_u}\overrightarrow{r_u} + 2u'(t)v'(t)\overrightarrow{r_u}\overrightarrow{r_v} + v'(t)^2\overrightarrow{r_v}\overrightarrow{r_v}}\,dt$$

On $\overrightarrow{r_u}$ i $\overrightarrow{r_v}$ són els respectius vectors unitaris de u(t) i v(t).

$dL^2 = (du\ \ dv)\begin{pmatrix} E & F \\ F & G \end{pmatrix}\begin{pmatrix} du \\ dv \end{pmatrix}$ on $E=\overrightarrow{r_u}\overrightarrow{r_u}$, $F=\overrightarrow{r_u}\overrightarrow{r_v}$, $G=\overrightarrow{r_v}\overrightarrow{r_v}$

i on $\vec{r} = \vec{u} + \vec{v}$ si $\vec{u}$ fos perpendicular a $\vec{v}$ el terme

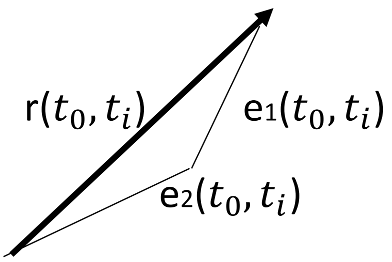
$2u'(t)v'(t)\overrightarrow{r_u}\overrightarrow{r_v}$ no existiria



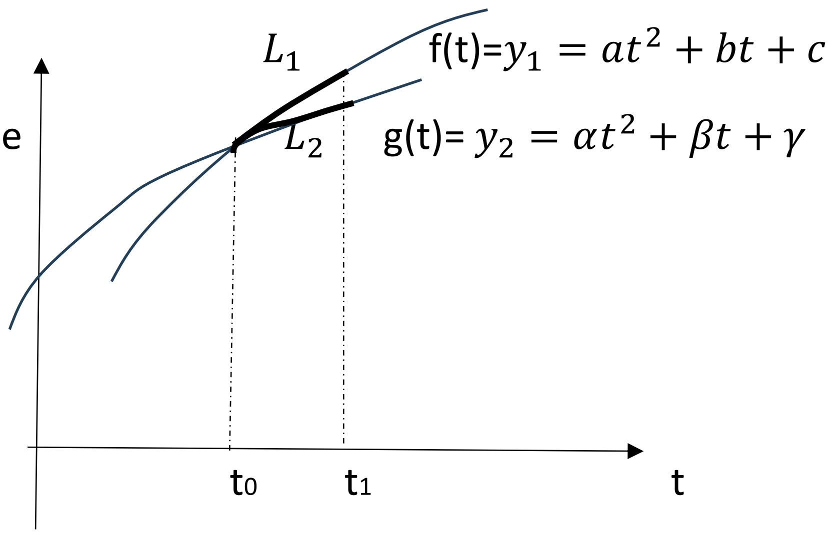
$e_1= v_1.t+ e_0$ on $e_0 + v_1.t_0 = v_2.t_0 + e_3$ i v_1= pendent d' e_1

$e_2= v_2.t+ e_3$ v_2= pendent d' e_2

quan $e_1(t_0, t_i)$ i $e_2(t_0, t_i)$ se sumen, obtenim $r(t_0, t_i)$:



i en e_1 i e_2 corves, succeeix el mateix:



$L_1 = \int_{t_0}^{t_1} \sqrt{1+(f'(t))^2}\, dt$= $\int_{t_0}^{t_1} \sqrt{1+(2at+b)^2} dt =$

(a=1, b=3 , c=2 , t_0=3 , t_1=4)

=$\int_{t_0}^{t_1} \sqrt{1+9+12t+4t^2} dt$=$\int_{t_0}^{t_1} \sqrt{10+12t+4t^2} dt$

Segons el “mètode Alemán”:

$\frac{10+12t+4t^2}{\sqrt{10+12t+4t^2}}$= $\frac{d}{dt}((mt+n).\ \sqrt{10+12t+4t^2}$)+$\frac{\lambda}{\sqrt{10+12t+4t^2}} \rightarrow$

$\rightarrow\ m.\sqrt{10+12t+4t^2}$+(mt+n)(1/2).$\frac{(8t+12)}{\sqrt{10+12t+4t^2}}$+$\frac{\lambda}{\sqrt{10+12t+4t^2}}$

$\rightarrow m.(10+12t+4t^2)$+(mt+n)($4t+6$) + λ=$10+12t+4t^2$

m=1/2 , n=3/4 , λ=-9/2

$\rightarrow \sqrt{10+12t+4t^2}$=

$= \frac{d}{dt}(\left(\frac{1}{2}\right) t + (\frac{3}{4}).\ \sqrt{10+12t+4t^2}$)+$\frac{(-\frac{9}{2})}{\sqrt{10+12t+4t^2}}$

$[10+12t+4t^2$=$(2t+3)^2+1]$

Integrem els 2 costats de la igualtat i la part $\int_{t_0}^{t_1} \frac{(-\frac{9}{2})}{\sqrt{(2t+3)^2+1}}$dt=

$\rightarrow$= (-9/2).(1/2).ArcSinh(2t+3)$\Big|_{t_0}^{t_1}$

Per tant: [[$\left(\frac{1}{2}\right) t + (\frac{3}{4})$]. $\sqrt{10+12t+4t^2}$] $\Big|_3^4$+

+(-9/2).(1/2).ArcSinh(2t+3)$\Big|_3^4$