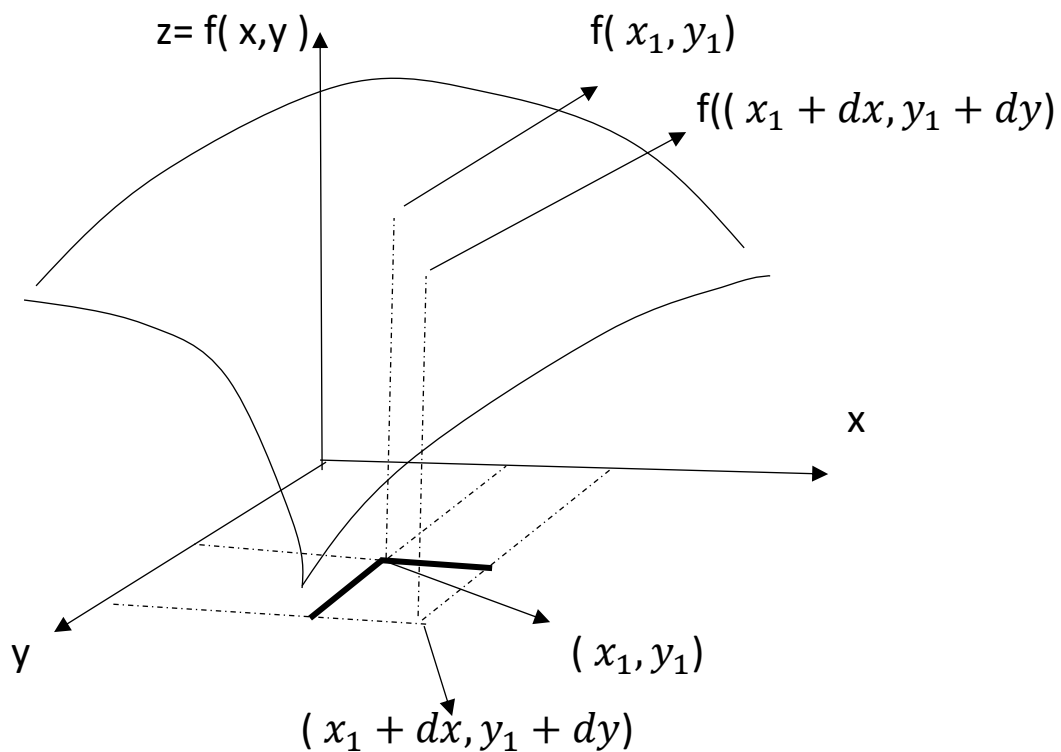


Taylor Series in partial derivatives:

in 2 variables:



According to Taylor: $\sum_{n=0}^{\infty} \frac{x^n \cdot f^n(a)}{n!}$

And the definition of derivative:

$$F'(x_1, y_1) = \lim_{\Delta p \rightarrow 0} \frac{f(x_1 + dx, y_1 + dy) - f(x_1, y_1)}{\Delta p} \text{ where } \Delta p = \sqrt{dx_1^2 + dy_1^2}$$

Depending of the value of "n" the slope will reach zero or ∞ .

the order of the

derivative



$$f^n(x_1, y_1) \cdot \Delta p$$

$$\begin{aligned} f^n(x_1, y_1) \cdot \Delta p = & \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{1 \cdot \partial^2 f}{2! \cdot \partial x^2} dx^2 + \frac{2 \cdot \partial^2 f}{2! \cdot \partial x \partial y} dx dy + \\ & + \frac{1 \cdot \partial^2 f}{2! \cdot \partial y^2} dy^2 + \frac{1 \cdot \partial^3 f}{3! \cdot \partial x^3} dx^3 + \frac{3 \cdot 2 \cdot \partial^3 f}{3! \cdot \partial x^2 \partial y} dx^2 dy^1 + \\ & + \frac{3 \cdot 2 \cdot \partial^3 f}{3! \cdot \partial x \partial y^2} dx^1 dy^2 + \frac{1 \cdot \partial^3 f}{3! \cdot \partial y^3} dy^3 + \dots \end{aligned}$$

$$\text{Which gives : } \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) \rightarrow 2 \cdot \frac{\partial^2 f}{\partial x \partial y}$$

(SCHWARZ'S THEOREM)

$$\begin{aligned} \text{And the same : } \frac{\partial}{\partial x} \left(2 \cdot \frac{\partial^2 f}{\partial x \partial y} \right) & \rightarrow 2 \cdot \frac{\partial^3 f}{\partial x \partial x \partial y} = 2 \cdot \frac{\partial^3 f}{\partial x \partial y \partial x} = 2 \cdot \frac{\partial^3 f}{\partial y \partial x \partial x} \rightarrow \\ & \rightarrow 3 \cdot 2 \cdot \frac{\partial^3 f}{3! \cdot \partial x^2 \partial y} \end{aligned}$$

And with $\frac{3 \cdot 2 \cdot \partial^3 f}{3! \cdot \partial x \partial y^2}$ produces the same mechanism .

Watching out :

$$x^n \equiv dx^n, dx^{n-1} dy^1, dx^{n-2} dy^2,$$

$$\text{Till } dx^1 dy^{n-1} \text{ and } dy^n$$