

Calculation of the volumes from n=3 to n=m:

$$V_3 \quad n=3 \quad x^2 + y^2 + z^2 \leq R^2$$

$$V_4 \quad n=4 \quad x^2 + y^2 + z^2 + t^2 \leq R^2$$

...

$$n \geq 4: \int_0^R (\sqrt{R^2 - t^2})^n dt \stackrel{R^n}{=} \frac{R^n}{2} \cdot \int_0^1 s^{1/2} \cdot (1-s)^{n/2} ds =$$

change of variable $t=R \cdot \sqrt{s}$

$$= \int_0^1 t^a (1-t)^b dt = \frac{\Gamma(1+a) \cdot \Gamma(1+b)}{\Gamma(2+a+b)}$$

On the other hand, to find $V_n = \frac{(\sqrt{\pi})^n \cdot (\frac{1}{n})(R)^n}{(\frac{1}{2}) \cdot \Gamma(\frac{n}{2})}$ (obtained from

of one method that I won't go into now but which is very beautiful)

you need to know that:

$$\int_{-\infty}^{\infty} e^{-r^2} r^{n-1} dr \stackrel{\uparrow}{=} \int_{-\infty}^{\infty} e^{-v} v^{(1/2)(n-1)} \left(\frac{1}{2}\right) \cdot v^{-1/2} dv \rightarrow$$

change of variable $v=r^2$, $r=v^{1/2}$, $dr=(1/2) \cdot v^{-1/2} dv$

$$\rightarrow \left(\frac{1}{2}\right) \cdot \int_{-\infty}^{\infty} e^{-v} v^{\left(\frac{1}{2}\right)(n-1)-(1/2)} dv = \left(\frac{1}{2}\right) \cdot \int_{-\infty}^{\infty} e^{-v} v^{\left(\frac{1}{2}\right)n-1} dv \rightarrow$$

$\rightarrow$ so that: $\left(\frac{1}{2}\right) \cdot \Gamma(n/2)$