

Demostració de: “Integral pel mètode Alemany” (I) i “Integral per parts” (II):

$$(I): \int_{x_1}^{x_2} \sqrt{a \cdot x^2 + b \cdot x + c} \cdot dx \quad \text{fem-ho fàcil: } \int_{x_1}^{x_2} \sqrt{x^2} \cdot dx$$

$$\int_{x_1}^{x_2} \sqrt{x^2} \frac{\sqrt{x^2}}{\sqrt{x^2}} \cdot dx = \int_{x_1}^{x_2} \frac{x^2}{\sqrt{x^2}} \cdot dx.$$

$$\frac{x^2}{\sqrt{x^2}} = \frac{d}{dx} [(mx + n) \cdot \sqrt{x^2}] + \frac{\lambda}{\sqrt{x^2}} = [m \cdot \sqrt{x^2} + (mx + n) \cdot (\frac{1}{2} \cdot \frac{2x}{\sqrt{x^2}})] + \frac{\lambda}{\sqrt{x^2}}$$

$$\rightarrow x = m \cdot x + (mx + n) \cdot 1 + \frac{\lambda}{x} \rightarrow x = mx + mx + n + \frac{\lambda}{x} \rightarrow$$

$$x^2 = m \cdot x^2 + m \cdot x^2 + nx + \lambda \quad 2m=1, \quad m=1/2 \quad n=0 \quad \lambda=0$$

Ara, suposem $x_2=2$ i $x_1=1$

$$\text{Ara: } \int_1^2 \frac{x^2}{\sqrt{x^2}} dx = \int_1^2 x \cdot dx = \left[\frac{1}{2} x \cdot \sqrt{x^2} \right]_1^2 + \int_1^2 \frac{0}{\sqrt{x^2}} dx \rightarrow$$

$$\rightarrow \left[\frac{x^2}{2} \right]_1^2 = \frac{1}{2} \cdot x^2 \Big|_1^2 + 0 \rightarrow 2 - (1/2) = 2 - (1/2): \text{ queda demostrat}$$

$$(II): \int (f \cdot g)' = \int f' \cdot g + \int f \cdot g' \rightarrow \int f \cdot g' = f \cdot g - \int f' \cdot g$$