

## SÈRIES DE TAYLOR, TANGENT(X):

Sabem que

$$P(x)=f(a) + f'(a).\frac{x-a}{1!} + f''(a).\frac{(x-a)^2}{2!} + \dots \\ + f^n(a).(x-a)^n/n!$$

I la sèrie sinus d'x és igual a:

$$\text{Sin}x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

Mentre que la Cosinus d'x:

$$\text{Cos}x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Per a qualsevol x i considerant els valors màxims possibles:

$$\text{Sin } 90 = 1 \text{ i } \text{Cos } 0 = 1.$$

També passa amb la tangent d'x: Sèrie de Taylor:

Coneguent que  $a \neq 0$

$$\text{tg}(a) = \text{tg}(a)$$

$$\text{tg}'(a) = 1 + \text{tg}^2(a)$$

$$\text{tg}''(a) = 2\text{tg}(a) + 2\text{tg}^3(a)$$

$$\text{tg}'''(a) = 2 + 8\text{tg}^2(a) + 6\text{tg}^4(a)$$

$$\text{tg}^{\text{IV}}(a) = 16\text{tg}(a) + 40\text{tg}^3(a) + 24\text{tg}^5(a)$$

...

$$\begin{aligned}
& \text{A més, } \operatorname{tg}(a) = (\operatorname{tg}^0) \cdot (x-a)^0 / 0! + \\
& + (1 + \operatorname{tg}^2(a)) \cdot (x-a) / 1! + (2\operatorname{tg}(a) + 2\operatorname{tg}^3(a)) \cdot (x-a)^2 / 2! \\
& + (2 + 8\operatorname{tg}^2(a) + 6\operatorname{tg}^4(a)) \cdot (x-a)^3 / 3! + \\
& + [16\operatorname{tg}(a) + 40\operatorname{tg}^3(a) + 24\operatorname{tg}^5(a)] \cdot (x-a)^4 / 4! \\
& + [16 + 16\operatorname{tg}^2(a) + 120\operatorname{tg}^2(a) + \\
& 120\operatorname{tg}^4(a) + 120\operatorname{tg}^4(a) + 5\operatorname{tg}^6(a)] \cdot \frac{(x-a)^5}{5!} + \dots
\end{aligned}$$

Ara pararem atenció calculant el **residu (remainder):**  $R_{n+1}(x)$ :

$$f(x) = P_n(x) + R_{n+1}(x)$$

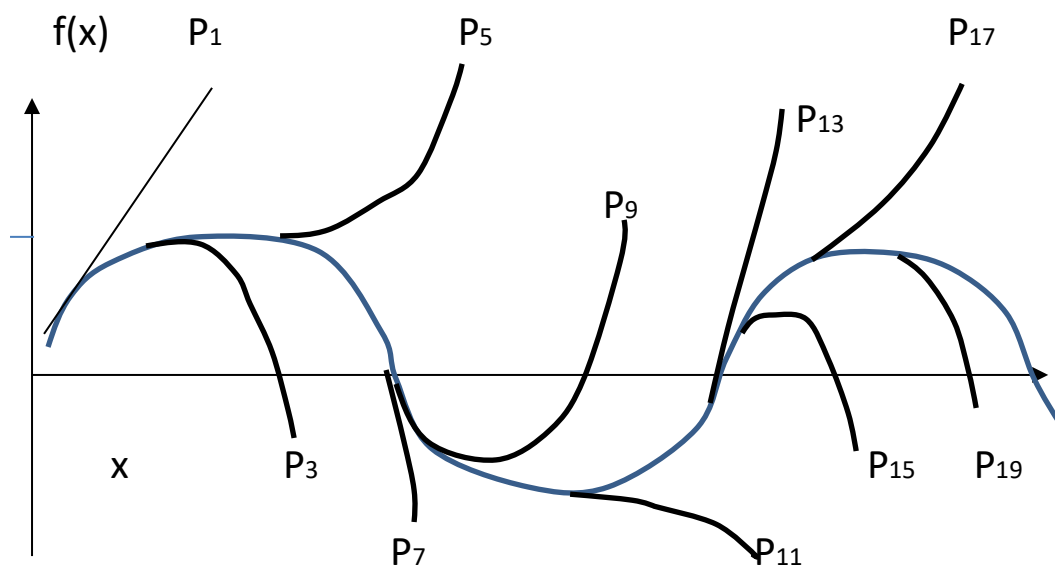
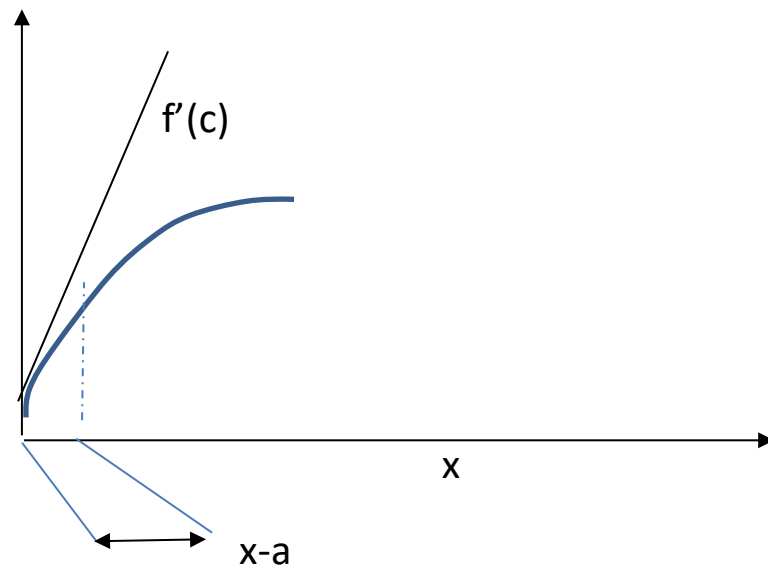
$$R_{n+1}(x) = \frac{f^{n+1}(c)}{(n+1)!} (x-a)^{n+1} \quad \text{on } c \in (a, x)$$

$$\text{És lògic que } f'(c) = \frac{f(x) - f(a)}{(x-a)} \text{ i } = \frac{R_n(x)}{(x-a)^n} = \frac{f^n(c)}{n!}$$

$$\text{mentre } x \text{ s'apropa a } \rightarrow a \text{ quan } n=1, \lim_{x \rightarrow a} \frac{R_n(x)}{(x-a)^n} = \frac{f^n(c)}{n!} = \frac{f(x) - f(a)}{(x-a)}$$

$$\frac{R_1(x)}{(x-a)^1} = \frac{f(x) - f(a)}{(x-a)} \rightarrow R_1(x) = f(x) - f(a) \rightarrow 0$$

$$n=0 \quad \lim_{x \rightarrow a} \frac{f'(c)}{(n+1)!} (x-a)^{n+1} = 0$$



El residuo a  $f^{(n+1)}(c)$  is  $\leq M$ , so  $R_{n+1}(x) \leq M \left| \frac{(x-a)^{n+1}}{(n+1)!} \right|$

Ara aprofundirem en el seu càlcul:

En  $f(x) = \sqrt[3]{x}$ , estimarem el seu càlcul quan  $x = 28$  usant la Sèrie de Taylor fins a  $n=2$ :

$$f(x) = x^{1/3}$$

$$f^{n=1}(x) = 1/3 \cdot x^{-2/3} \rightarrow f^{n=1}(27) = 0'037037$$

$$f^{n=2}(x) = -2/9 \cdot x^{-5/3} \rightarrow f^{n=2}(27) = 0'000914$$

$$\text{per tant si } a=27: \sqrt[3]{x} = f(27) + f^{n=1}(27) \cdot (x-27)/1! + f^{n=2}(27) \cdot (x-27)^2/2! + R_{n>2}(27)$$

$$\text{ja que: } x=28, \sqrt[3]{28} = 3'036579.$$

$$f(28) = P_n(28) + R_{n+1}(28)$$

$$\text{o el que és el mateix: } R_3(28) = \frac{f^{n=3}(c)}{3!} (28 - 27)^3 \text{ and } c=28.$$

$$f^{n=3}(28) = 1/3 \cdot (28)^{-2/3}$$

$$\text{tornant a } tg(x) = [(1/2) + (2/6) + (16/120)] + [1 \cdot (x-a)^2 + \\ + \left(\frac{16}{24}\right)(x-a)^4 tg(a)] + \left[\left(\frac{1}{2}\right)(x-a)^1 + \left(\frac{8}{6}\right)(x-a)^3 + \right.$$

$$\left.\left(\frac{136}{120}\right)(x-a)^5 \cdot tg(a)^2\right] + [1(x-a)^2 + 10(x-a)^4 \cdot tg(a)^3] +$$

$$+ [1(x-a)^3 + \left(\frac{240}{120}\right)(x-a)^5 \cdot tg(a)^4] +$$

$$+ \left[\left(\frac{24}{4}\right)(x-a)^4 \cdot tg(a)^5\right] + \dots$$

Una altra versió de tg(x) és:

$$\operatorname{tg}(x) = \frac{\sin(x)}{\cos(x)} = \frac{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}}{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}}$$

podem representar Sin(x) i Cos(x) com a producte de binomis, lògicament segons les arrels que tenen (segueixo el sistema d'Euler):

$$\sin(x) = C_1 \cdot x \cdot (x + \pi) (x - \pi) (x + 2\pi) (x - 2\pi) (x + 3\pi) (x - 3\pi) \dots$$

$$\cos(x) = C_2 \cdot \left(x + \frac{\pi}{2}\right) \left(x - \frac{\pi}{2}\right) \left(x + \frac{3\pi}{2}\right) \left(x - \frac{3\pi}{2}\right) \dots$$

$$\cdot \left(x + \frac{5\pi}{2}\right) \left(x - \frac{5\pi}{2}\right) \dots$$

$$\operatorname{tg}(x) = \frac{C_1 (x^2 - \pi^2) (x^2 - (2\pi)^2) (x^2 - (3\pi)^2) (x^2 - (4\pi)^2) \dots}{C_2 \left(x^2 - \left(\frac{\pi}{2}\right)^2\right) (x^2 - (3\pi/2)^2) (x^2 - (5\pi/2)^2) \dots}$$

$$\text{segons Taylor, el } \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots =$$

$$= C_1 \cdot x \cdot (x + \pi) (x - \pi) (x + 2\pi) (x - 2\pi) (x + 3\pi) (x - 3\pi) \dots$$

Si dividim els 2 costats de la igualtat per x:

$$1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \frac{x^8}{9!} - \dots = C_1 (x^2 - \pi^2) (x^2 - (2\pi)^2) \dots$$

$$\cdot (x^2 - (3\pi)^2) (x^2 - (4\pi)^2) \dots$$

$$\text{i si } x=0: 1 = C_1 (-\pi^2) (-4\pi^2) (-9\pi^2) \dots =$$

$$= \prod_{n=0}^{\infty} C_1 \cdot (-1)^n (\pi^2)^n n^2$$

si també usem x=0 en Cos(x):

$$\cos(x) = C_2 \left(-\left(\frac{\pi}{2}\right)^2\right) \left(-(3\pi/2)^2\right) \left(-(5\pi/2)^2\right) \dots$$

$$\text{O sigui: } \cos(x) = \prod_{n=0}^{\infty} C_2 \cdot (-1)^n (\pi^2)^n (2n+1)^2$$

$$\text{Per tant: } \text{tg}(x) = \prod_{n=0}^{\infty} \frac{c_1 \cdot (n)^2 \cdot (4)^n}{c_2 \cdot (2n+1)^2}$$