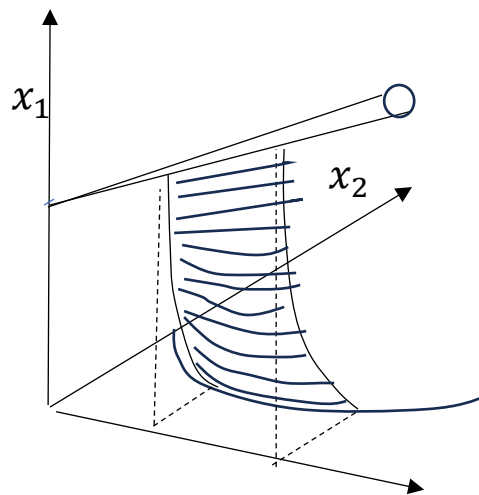


Surfaces :

$$x_1 = \alpha_1 \cdot t + \alpha_2$$

$$x_2 = \beta_1 \cdot t^2 + \beta_2 \cdot t + \beta_3$$



about

$$\sqrt{dS^2} = \sqrt{dt^2 + (dx_1)^2 + (dx_2)^2} \frac{dt}{dt} =$$

$= \sqrt{1 + \left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2} \cdot dt$ and now we will use the method German to solve :

$$\int_a^b dS = \int_a^b \sqrt{1 + (\alpha_1)^2 + (2\beta_1 \cdot t + \beta_2)^2} \cdot dt =$$

$$= \int_a^b \sqrt{1 + (\alpha_1)^2 + (4 \cdot \beta_1^2 \cdot t^2 + \beta_2^2 + 4 \cdot \beta_1 \beta_2 \cdot t)} \cdot dt =$$

Where $w = 4 \cdot \beta_1^2$, $v = 4 \cdot \beta_1 \beta_2$, $u = 1 + (\alpha_1)^2 + \beta_2^2$

$$\text{and } \int_a^b dS = \int_a^b \sqrt{w \cdot t^2 + v \cdot t + u} \cdot dt$$

$$\sqrt{w \cdot t^2 + v \cdot t + u} \frac{\sqrt{w \cdot t^2 + v \cdot t + u}}{\sqrt{w \cdot t^2 + v \cdot t + u}} = \frac{d}{dt} \left[(r \cdot t + s) \sqrt{w \cdot t^2 + v \cdot t + u} \right] + \frac{\lambda}{\sqrt{w \cdot t^2 + v \cdot t + u}}$$

Where is it seen? clear that Numerator/Denominator= Dividend /Divisor=

= (Quotient x Divisor + Remainder/ Divisor)

$$w.t^2 + v.t + u = r(w.t^2 + v.t + u) + (rt+s). \frac{1}{2}. [(2tw + v)] + \lambda$$

That, operating correctly , give as a result :

$$r = \frac{1}{2}, s = \frac{1}{4} \cdot \frac{v}{w}, \quad \lambda = \frac{1}{2}u - \frac{v^2}{4w}$$

$$\text{and now: } \int_a^b \sqrt{w.t^2 + v.t + u} . dt =$$

$$= \left(\frac{1}{2} \cdot t + \frac{1}{4} \cdot \frac{v}{w} \right) \cdot \sqrt{w.t^2 + v.t + u} \Big|_a^b + \int_a^b \frac{\left(\frac{1}{2}u - \frac{v^2}{4w} \right) dt}{\sqrt{w.t^2 + v.t + u}}$$

where this last integral is :

$$\text{Assuming : } \alpha_1 = 4 \quad \beta_1 = 3, \quad \beta_2 = 2 \rightarrow w = 36, v = 24, u = 21$$

$$\int_a^b \frac{\left(\frac{1}{2}u - \frac{v^2}{4w} \right)}{\sqrt{(6t+2)^2 + 17} \frac{\sqrt{17}}{\sqrt{17}}} dt = \int_a^b \frac{\left(\frac{1}{2}u - \frac{v^2}{4w} \right) dt}{\sqrt{\left(\frac{6}{\sqrt{17}}t + \frac{2}{\sqrt{17}} \right)^2 + 1} \frac{\sqrt{17}}{1}} =$$

$$\left(\frac{1}{2}u - \frac{v^2}{4w} \right) \cdot \frac{1}{\frac{6}{\sqrt{17}}} \cdot \frac{1}{\sqrt{17}} \cdot \text{ArcSinh} \left(\frac{6}{\sqrt{17}}t + \frac{2}{\sqrt{17}} \right) \Big|_a^b$$

Finally :

$$\begin{aligned} \int_a^b \sqrt{w.t^2 + v.t + u} . dt &= \left(\frac{1}{2} \cdot t + \frac{1}{4} \cdot \frac{v}{w} \right) \cdot \sqrt{w.t^2 + v.t + u} \Big|_a^b + \\ &+ \left(\frac{1}{2}u - \frac{v^2}{4w} \right) \cdot \frac{1}{\frac{6}{\sqrt{17}}} \cdot \frac{1}{\sqrt{17}} \cdot \text{ArcSinh} \left(\frac{6}{\sqrt{17}}t + \frac{2}{\sqrt{17}} \right) \Big|_a^b \end{aligned}$$