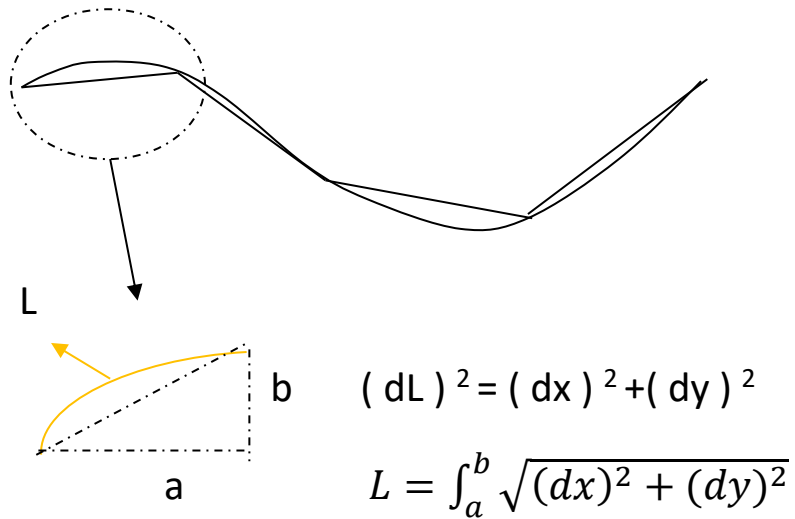


ARCLENGTH:

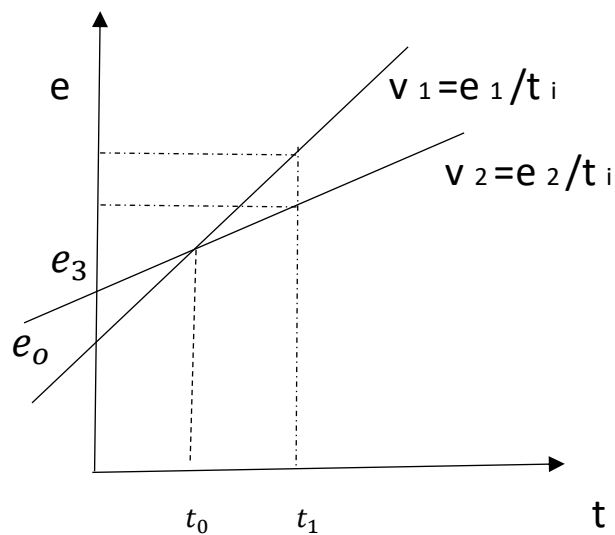


and knowing the definition of derivative: " $f'(x) = dy / dx$ "

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$x(t) = t \text{ and } y(t)$$

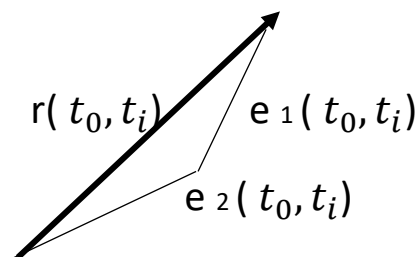


$$e_1 = v_1 \cdot t + e_0 \quad \text{where} \quad e_0 + v_1 \cdot t_0 = v_2 \cdot t_0 + e_3$$

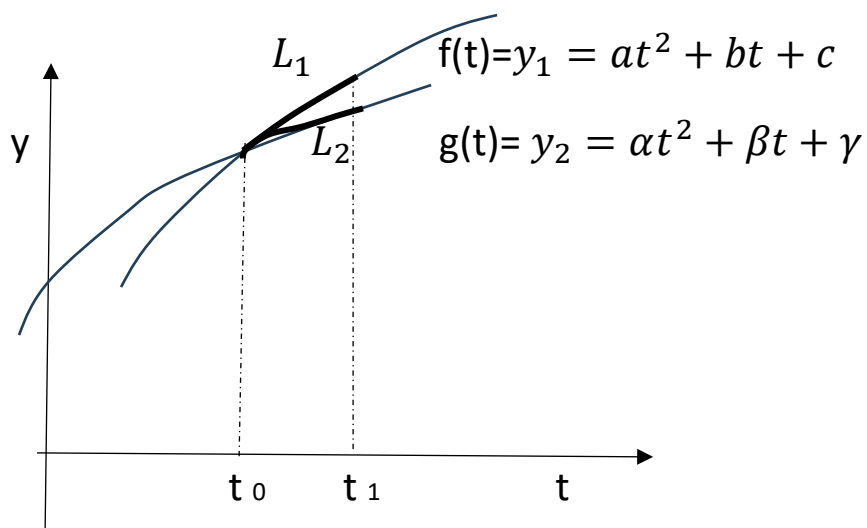
and $v_1 = \text{slope of } e_1$

$$e_2 = v_2 \cdot t + e_3 \quad e_3 = v_2 \cdot t_0 \quad v_2 = \text{slope of } e_2$$

when $e_1(t_0, t_i)$ and $e_2(t_0, t_i)$ are added, we get $r(t_0, t_i)$:



and in e_1 and e_2 curves, the same thing happens :



$$L_1 = \int_{t_0}^{t_1} \sqrt{1 + (f'(t))^2} dt = \int_{t_0}^{t_1} \sqrt{1 + (2at + b)^2} dt =$$

$$(a = 1, b = 3, c = 2, t_0 = 3, t_1 = 4)$$

$$= \int_{t_0}^{t_1} \sqrt{1 + 9 + 12t + 4t^2} dt = \int_{t_0}^{t_1} \sqrt{10 + 12t + 4t^2} dt$$

According to the " method German :

$$\frac{10+12t+4t^2}{\sqrt{10+12t+4t^2}} = \frac{d}{dt} ((mt + n) \cdot \sqrt{10 + 12t + 4t^2}) + \frac{\lambda}{\sqrt{10+12t+4t^2}} \rightarrow$$

$$\rightarrow m \cdot \sqrt{10 + 12t + 4t^2} + (mt + n) \cdot \frac{(8t+12)}{\sqrt{10+12t+4t^2}} + \frac{\lambda}{\sqrt{10+12t+4t^2}} \rightarrow$$

$$\rightarrow m \cdot (10 + 12t + 4t^2) + (mt + n)(4t + 6) + \lambda = 10 + 12t + 4t^2$$

$$m = 1/2, n = 3/4, \lambda = -9/2$$

$$\rightarrow \sqrt{10 + 12t + 4t^2} =$$

$$= \frac{d}{dt} \left(\left(\frac{1}{2} \right) t + \left(\frac{3}{4} \right) \cdot \sqrt{10 + 12t + 4t^2} \right) + \frac{\left(-\frac{9}{2} \right)}{\sqrt{10+12t+4t^2}}$$

$$[10 + 12t + 4t^2 = (2t + 3)^2 + 1]$$

We integrate the 2 sides of equality and the part

$$\int_{t_0}^{t_1} \frac{\left(-\frac{9}{2} \right)}{\sqrt{(2t+3)^2+1}} dt =$$

$$\rightarrow = \left(-\frac{9}{2} \right) \cdot \left(\frac{1}{2} \right) \cdot \text{ArcSinh}(2t+3) \Big|_{t_0}^{t_1}$$

$$\text{Therefore : } \left[\left(\frac{1}{2} \right) t + \left(\frac{3}{4} \right) \cdot \sqrt{10 + 12t + 4t^2} \right] \Big|_3^4 +$$

$$+ \left(-\frac{9}{2} \right) \cdot \left(\frac{1}{2} \right) \cdot \text{ArcSinh}(2t+3) \Big|_3^4$$