

Taylor series:

Suppose $f(x) = \frac{1}{1+x^2}$ $a = x^2$ if $a = 0$, then $x=0$

$$\sum_{n=0}^{\infty} \frac{x^n \cdot f^n(0)}{n!} = \sum_{n=0}^{\infty} \frac{x^n \cdot a_n}{n!}$$

Knowing that I have previously calculated the McLaurin SERIES (specific case of Taylor where $a=0$): $f(0) = 1$

$$f'(0) = \frac{-2x}{(1+x^2)^2} = 0 \quad f''(0) = \frac{-2}{(1+x^2)^2} + 0 \quad f'''(x) = 0$$

$$f^{IV}(0) = \frac{-8}{(1+x^2)^3} \quad f^V(0) = 0 \quad f^{VI}(0) = \frac{-48}{(1+x^2)^4}$$

$$f^{VII}(0) = 0 \quad f^{VIII}(0) = \frac{-384}{(1+x^2)^5} \quad f^{IX}(0) = 0$$

$$f^X(0) = \frac{-3840}{(1+x^2)^6} \dots$$

$$f^{2n}(a) = f^{2n}(0) = f^n(0) \cdot (-2 \cdot n)$$

that is : $a_{2n} = a_0 \cdot (-2n)$

$$a_0 = 1 \quad a_2 = -2 \quad a_4 = -8 \quad a_6 = -48 \quad a_8 = -384 \quad a_{10} = -3840$$

...

$$f(x) = \frac{1}{1+x^2} = 1 + \sum_{n=1}^{\infty} \frac{x^{2n} \cdot a_{2n}}{2n!} = 1 - \frac{x^2 \cdot a_2}{2!} - \frac{x^4 \cdot a_4}{4!} - \frac{x^6 \cdot a_6}{6!} - \frac{x^8 \cdot a_8}{8!} - \dots$$