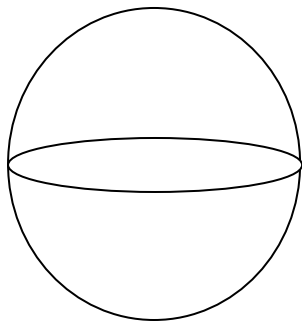
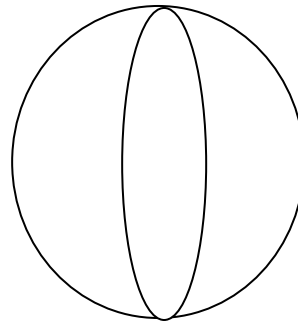


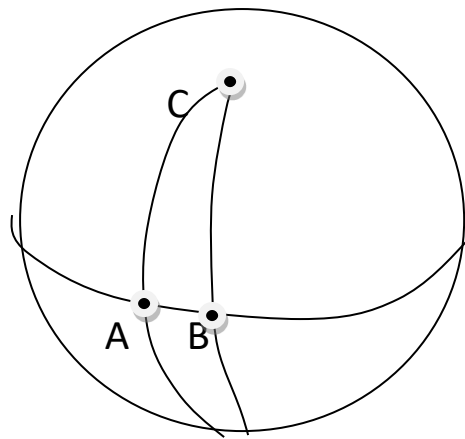
RICCI, CURVATURE, RIEMANN TENSORS...



LONGITUDE



LATITUDE



Let's assume only 2-D: A): scale factors: $x_1, x_2 \rightarrow x_\lambda \quad \lambda = 1, 2$

B): $\vec{u}_\mu = \sum_{\lambda=1}^2 \alpha_\mu^\lambda \cdot x_\lambda$ and $\vec{u}_\nu = \sum_{\lambda=1}^2 \alpha_\nu^\lambda \cdot x_\lambda$

Being $\vec{A} = A^i \cdot \vec{e}_i$ the "field" we are studying: $i=1, 2$

if we differentiate infinitesimally with respect to each of the 2 scale factors x_λ (called: $\widehat{x}_\lambda$): $\frac{d\vec{A}}{dx^\lambda} = \frac{dA^i}{dx^\lambda} \cdot \vec{e}_i + A^i \cdot \frac{d\vec{e}_i}{dx^\lambda}$

Note that $\frac{d\vec{e}_i}{dx^\lambda} = \Gamma_{i\lambda}^\beta \vec{s}_\beta$ where $\beta=1, 2$ and "s" is a unit vector.

Unit vectors are represented as scale factors.

$$\vec{e}_i = \alpha_i^\beta \cdot s_\beta = \vec{u}_\lambda$$

$$\text{Therefore: } \frac{d\vec{e}_i}{dx^\lambda} = \frac{d\alpha_i^\beta}{dx^\lambda} \cdot s_\beta = \Gamma_{i\lambda}^\beta \cdot s_\beta$$

The Ricci tensor or specific case of the Riemann tensor is represented as the remainder between B and A

$$\frac{dA_j^i}{dx^1} \vec{e}_1 - \frac{dA_j^i}{dx^2} \vec{e}_2 = R_{j\lambda}^i$$

A_j^i represents a matrix since the dimension is ≥ 2

$$(i:\text{rows}, j:\text{columns}). \text{ Example: } \begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix}$$

$$(d_1 d_2) \begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} a_1^1 \cdot e_1 + a_2^1 \cdot e_2 \\ a_1^2 \cdot e_1 + a_2^2 \cdot e_2 \end{pmatrix}$$

$$(d_1, d_2) \text{ is the partial derivative: } \left(\frac{d}{dx^1}, \frac{d}{dx^2} \right)$$

$$(d_1 a_1^1 + d_2 a_1^2, d_1 a_2^1 + d_2 a_2^2) \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = [(d_1 a_1^1 + d_2 a_1^2) e_1 +$$

$$+ (d_1 a_2^1 + d_2 a_2^2) e_2] = \frac{da_1^1}{dx^1} e_1 + \frac{da_1^2}{dx^2} e_1 + \frac{da_2^1}{dx^1} e_2 + \frac{da_2^2}{dx^2} e_2$$

$$\text{Where } e_1 = \vec{e}_1$$

$$R_{j\lambda\beta}^i = \frac{dR_{j\lambda}^i}{dx^\lambda} = \frac{d}{dx^\lambda} \left[\frac{dA_j^i}{dx^1} \vec{e}_1 - \frac{dA_j^i}{dx^2} \vec{e}_2 \right] = d_1 (\Gamma_{j1}^i) \cdot \vec{e}_1 + \Gamma_{j1}^i (\Gamma_{i1}^\beta \cdot s_\beta) -$$

$$- d_2 (\Gamma_{j2}^i) \cdot \vec{e}_2 - \Gamma_{j2}^i (\Gamma_{i2}^\beta \cdot s_\beta) \text{ when } \vec{e}_i = \alpha_i^\beta \cdot s_\beta \text{ and } \lambda = 1, 2$$

$$R_{j\lambda\beta}^i = d_1 (\Gamma_{j1}^i) \cdot (\alpha_1^\beta \cdot s_\beta) + \Gamma_{j1}^i (\Gamma_{i1}^\beta \cdot s_\beta) - d_2 (\Gamma_{j2}^i) \cdot \alpha_2^\beta \cdot s_\beta -$$

$$- \Gamma_{j2}^i (\Gamma_{i2}^\beta \cdot s_\beta)$$