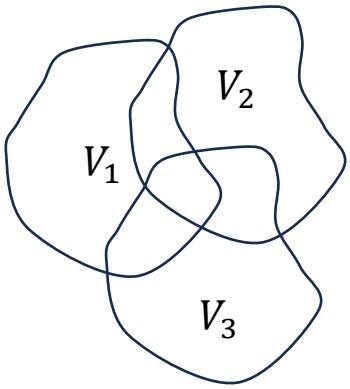
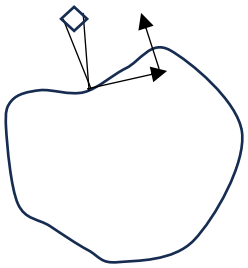
SET THEORY:



$V_1 \cup V_2 \cup V_3 - V_1 \cap V_2 - V_1 \cap V_3 - V_2 \cap V_3 -$
$-V_1 \cap V_2 \cap V_3$

Once you know how to calculate length, now it's time for surface area and volume :

AREA



let's remember **arclength** when the triangle is not a rectangle :

u(t) , t

on **the surface** we will resort to DE SITTER **:**

2- D , d S^2=d $x_0^2 + \sum_{i=1}^{2} dx_i^2$ n=2 where d x_0= - ct

and x_1(t), x_2(t),

Pay attention to the negative sign of d x_0= -c. t (Minkowski dictated it that way)

We , however , do not we will take into account .

S= $\int_a^b \sqrt{1 + (\frac{dx_1(t)}{dt})^2 + (\frac{dx_2(t)}{dt})^2}dt$

[assuming that c= speed of light =1]

On **surfaces** :

we will call this one 3x3 matrix " M" and($t\ dx_1\ dx_2$)

and the vectors units of each$\overrightarrow{dx_n}$ will be:, $\widehat{u_0}$, $\widehat{u_1}$, $\widehat{u_2}$

dS ^{2} = $(t \quad dx_1 \quad dx_2)$."M".$\begin{pmatrix} t \\ dx_1 \\ dx_2 \end{pmatrix}$

"M":$\begin{pmatrix} \widehat{u_0}^2 & \widehat{u_0}\widehat{u_1} & \widehat{u_0}\widehat{u_2} \\ \widehat{u_1}\widehat{u_0} & \widehat{u_1}^2 & \widehat{u_1}\widehat{u_2} \\ \widehat{u_2}\widehat{u_0} & \widehat{u_2}\widehat{u_1} & \widehat{u_2}^2 \end{pmatrix}$

Where $\widehat{u_0}$ $\widehat{u_1}$, $\widehat{u_2}$they do not form right angles with each other.

Operating , we find

dS ^{2} = $t^2.\widehat{u_0}^2 + 2t.dx_1\widehat{u_1}\widehat{u_0}$+2t. $dx_2\widehat{u_0}\widehat{u_2}$+2 $dx_1dx_2\widehat{u_1}\widehat{u_2}$+

$+dx_1^2\hat{u}_1^2 + dx_2^2\hat{u}_2^2$

While for **Volumes** we add another variable: x 3 (t, φ) therefore 3-D. Let's assume that as an example: x_0 (t)= t

and the vectors unitary of $\overrightarrow{dx_n}$: are: $\hat{u}_0, \hat{u}_1, \hat{u}_2, \hat{u}_3$ x_1 (t)= αt

which are not ⊥ between them. x_2 (t)= β t

x_3 (t, φ)= γt φ

all depends on a value of a determined φ.

The "M" will be 4x4

d V^2=$(t\ dx_1\ dx_2\ dx_3)\begin{pmatrix} \hat{u}_o^2 & \cdots & \hat{u}_0\hat{u}_3 \\ \vdots & \ddots & \vdots \\ \hat{u}_3\hat{u}_0 & \cdots & \hat{u}_3^2 \end{pmatrix}(t\ dx_1\ dx_2\ dx_3)^T$

$$V=\int_a^b \sqrt{1+(\frac{dx_1(t)}{dt})^2+(\frac{dx_2(t)}{dt})^2+(\frac{dx_3(t)}{dt})^2}\,dt$$

Since according to De Sitter: $dV^2 = dx_0^2 + \sum_{i=1}^{3}(dx_i)^2$

$$dV^2 = t^2\hat{u}_0^2 + 2t.dx_1\hat{u}_0\hat{u}_1 + 2t.dx_2\hat{u}_0\hat{u}_2 + 2t.dx_3\hat{u}_3\hat{u}_0 + 2.dx_2dx_1\hat{u}_2\hat{u}_1 + dx_1^2\hat{u}_1^2 + dx_2^2\hat{u}_2^2 + 2.dx_3dx_1\hat{u}_3\hat{u}_1 + 2.dx_2dx_3\hat{u}_3\hat{u}_2 + dx_3^2\hat{u}_3^2$$