Study of the metric tensor:

Through it we know that we can calculate the length :

$L=\int_a^b \sqrt{g_{ij}.\dot{x}^i.\dot{x}^j}\, dt$

when i=j, Kronecker's delta $=e^i.e_i = \delta_i^i = 1$

Then g_{ij}= components of the metric (which can also be represented as a matrix: $\begin{pmatrix} g_{ii} & g_{ji} \\ g_{ij} & g_{jj} \end{pmatrix}$in the 2-D case.

Metrics don't have to be numbers.

Returning to the Length L: $\frac{dx^i}{dt}= v^i$, $\frac{dx^j}{dt}= v^j$when i=j= 1.2

$\vec{v}=(v_1\overrightarrow{e_1} + v_2\overrightarrow{e_2})$

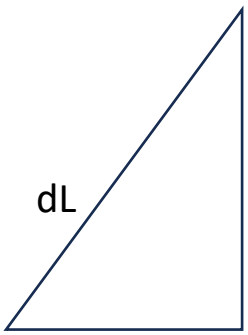
$\dot{x}^i.\dot{x}^j.\, dt^2= \|\vec{v}\|^2=(v_1\overrightarrow{e_1} + v_2\overrightarrow{e_2}).(v_1\overrightarrow{e_1} + v_2\overrightarrow{e_2})=$

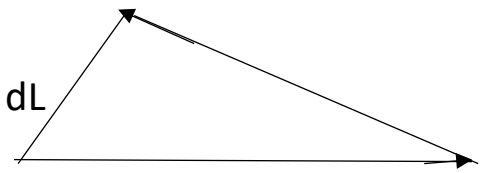
$g_{ii}.\, v_1v_1.\overrightarrow{e_1}.\overrightarrow{e_1}+ g_{ij}.\, v_1v_2.\overrightarrow{e_1}.\overrightarrow{e_2} + g_{ji}.\, v_2v_1.\overrightarrow{e_2}.\overrightarrow{e_1} +$

$+ g_{jj}.\, v_2v_2.\overrightarrow{e_2}.\overrightarrow{e_2}$

Since and are canonical and linearly independent $\overrightarrow{e_1}$bases $\overrightarrow{e_2}$, we use the Kronecker delta.

In 2-D:





the theorem of Pythagoras, does work

the theorem does not work

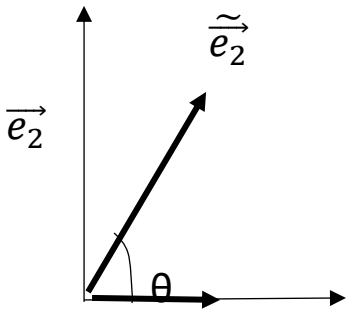
therefore: $dL^2 = g_{00}dx^0dx^0 + g_{01}dx^1dx^0 + g_{10}dx^0dx^1 + g_{11}dx^1dx^1$

we have what is called the Metric Matrix : $\begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix}$

in 3-D we can conceive of the surface:

$$dS^2 = (dx^1dx^2dx^3).\begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{bmatrix}.(dx^1dx^2dx^3)^T$$

Let's now calculate the Angle:



$\widetilde{e_1} = 1.\overrightarrow{e_1}, 0.\overrightarrow{e_2}$

$\widetilde{e_2}$=Body θ. $\overrightarrow{e_1}$, Sinθ . $\overrightarrow{e_2}$

$\overrightarrow{e_1}.\overrightarrow{e_1}$=1 , $\overrightarrow{e_2}.\overrightarrow{e_2}$= 1 , $\overrightarrow{e_2}.\overrightarrow{e_1}$=0

$\overrightarrow{e_1} = \widetilde{e_1}$where $\overrightarrow{e_1}$ i $\overrightarrow{e_2}$= unit vectors

We can understand that $\widetilde{e_1}.(\widetilde{e_2} - \widetilde{e_1})=0 \rightarrow \widetilde{e_1}\widetilde{e_2} - \widetilde{e_1}\widetilde{e_1}=0 \rightarrow$

$\rightarrow |\widetilde{e_1}\widetilde{e_2}| = \|\widetilde{e_1}\|^2$

As well as $\|\widetilde{e_1} + \widetilde{e_2}\|^2 = \|\widetilde{e_1}\|^2 + \|\widetilde{e_2}\|^2 + 2\|\widetilde{e_1}\|.\|\widetilde{e_2}\|$

Which expressed in canonical or “reference” coordinates: ($\overrightarrow{e_1}, \overrightarrow{e_2}$):

$\|\widetilde{e_1} + \widetilde{e_2}\|^2 = ((\overrightarrow{e_1} + \overrightarrow{e_1}.Cos\theta)^2 + (0 + \overrightarrow{e_2}.Sin\theta)^2 =$

$= [(\overrightarrow{e_1})^2 + 2.(\overrightarrow{e_1})^2.Cos\theta + (\overrightarrow{e_1})^2.(Cos\theta)^2] +$

$+(0 + \overrightarrow{e_2}.Sin\theta)^2$ (②)

$\left\|\widetilde{\overrightarrow{e_1}}\right\|^2 + \left\|\widetilde{\overrightarrow{e_2}}\right\|^2$+2 $\left\|\widetilde{\overrightarrow{e_1}}\right\|.\left\|\widetilde{\overrightarrow{e_2}}\right\|= (\overrightarrow{e_1}+0)^2 + [(\mathrm{Cos}\theta.\overrightarrow{e_1})^2 +$
$(\mathrm{Sin}\theta.\overrightarrow{e_2})^2]$+2.$\sqrt{(\overrightarrow{e_1})^2}.\sqrt{[(\mathrm{Cos}\theta.\overrightarrow{e_1})^2 + (\mathrm{Sin}\theta.\overrightarrow{e_2})^2} \rightarrow$

Since they are unit vectors,$(\overrightarrow{e_1})^2 = 1$, $(\overrightarrow{e_2})^2 = 1$

i $\overrightarrow{e_1}.\overrightarrow{e_2}$=0 and if$\vec{a} = (a_1, a_2)$ $\|\vec{a}\|$=$\sqrt{a_1^2 + a_2^2}$

→1+ 1 . $(\mathrm{Cos}\theta)^2$+1. $(\mathrm{Sin}\theta)^2$+2.$\sqrt{1.(\mathrm{Cos}\theta)^2 + 1.(\mathrm{Sin}\theta)^2}$ (①)

If we now equalize both sides of the equality: (②) = (①)

1+2.Body θ+ 1. $(\mathrm{Cos}\theta)^2$+1. $(\mathrm{Sin}\theta)^2$=1+1+2.1

$2(1+$ Cos θ)= 4

which coincides with the <u>triangular inequality:</u>

(1+Body θ) ≤2→ $\left|\widetilde{\overrightarrow{e_1}}\widetilde{\overrightarrow{e_2}}\right| \leq \left\|\widetilde{\overrightarrow{e_1}}\right\|.\left\|\widetilde{\overrightarrow{e_2}}\right\|$

And now: <u>Volume differential</u> :

$dV^2 = \sqrt{g_{00}g_{11}g_{22}g_{33}}.dx^0dx^1dx^2dx^3$