Estudi del tensor mètric:

Mitjançant ell sabem que podem calcular la longitud:

$L=\int_a^b \sqrt{g_{ij}.\dot{x}^i.\dot{x}^j}\,dt$

quan i=j, la delta de Kronecker= $e^i.e_i=\delta_i^i=1$

Aleshores g_{ij}= components de la mètrica (que també pot representar-se com a matriu: $\begin{pmatrix} g_{ii} & g_{ji} \\ g_{ij} & g_{jj} \end{pmatrix}$ en el cas de 2-D.

La mètrica no tenen perquè ser números.

Tornant a la Longitud L: $\frac{dx^i}{dt}=v^i$, $\frac{dx^j}{dt}=v^j$ quan i=j= 1,2

$\vec{v}=(v_1\overrightarrow{e_1}+v_2\overrightarrow{e_2})$

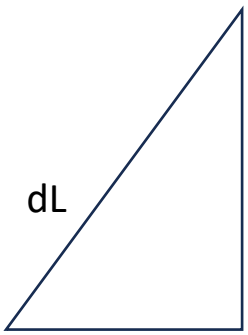
$\dot{x}^i.\dot{x}^j.dt^2=\|\vec{v}\|^2=(v_1\overrightarrow{e_1}+v_2\overrightarrow{e_2}).(v_1\overrightarrow{e_1}+v_2\overrightarrow{e_2})=$

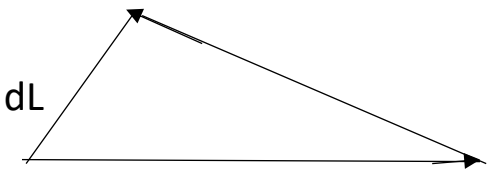
$g_{ii}.v_1v_1.\overrightarrow{e_1}.\overrightarrow{e_1}+g_{ij}.v_1v_2.\overrightarrow{e_1}.\overrightarrow{e_2}+g_{ji}.v_2v_1.\overrightarrow{e_2}.\overrightarrow{e_1}+$

$+g_{jj}.v_2v_2.\overrightarrow{e_2}.\overrightarrow{e_2}$

Al ser $\overrightarrow{e_1}$ i $\overrightarrow{e_2}$ bases canòniques i linealment independents, usem la delta de Kronecker.

En 2-D:





funciona el teorema de Pitàgores

no funciona el teorema de Pitàgores,

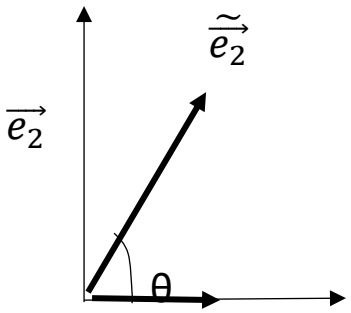
per tant: $dL^2=g_{00}dx^0dx^0+g_{01}dx^1dx^0+g_{10}dx^0dx^1+g_{11}dx^1dx^1$

tenim el que s'anomena la <u>Matriu Mètrica</u>: $\begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix}$

en 3-D podem concebre la superfície:

$$dS^2 = (dx^1 dx^2 dx^3).\begin{bmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{bmatrix}.(dx^1 dx^2 dx^3)^T$$

Passem ara a calcular <u>l'Angle:</u>



$\widetilde{\overrightarrow{e_1}} = 1.\overrightarrow{e_1}, 0.\overrightarrow{e_2}$

$\widetilde{\overrightarrow{e_2}}$ =Cosθ. $\overrightarrow{e_1}$, Sinθ. $\overrightarrow{e_2}$

$\overrightarrow{e_1}.\overrightarrow{e_1}$=1 , $\overrightarrow{e_2}.\overrightarrow{e_2}$=1 , $\overrightarrow{e_2}.\overrightarrow{e_1}$=0

$\overrightarrow{e_1} = \widetilde{\overrightarrow{e_1}}$ on $\overrightarrow{e_1}$ i $\overrightarrow{e_2}$= vectors unitaris

Podem entendre que $\widetilde{\overrightarrow{e_1}}.(\widetilde{\overrightarrow{e_2}} - \widetilde{\overrightarrow{e_1}})=0 \rightarrow \widetilde{\overrightarrow{e_1}}\widetilde{\overrightarrow{e_2}} - \widetilde{\overrightarrow{e_1}}\widetilde{\overrightarrow{e_1}} =0 \rightarrow$

$\rightarrow |\widetilde{\overrightarrow{e_1}}\widetilde{\overrightarrow{e_2}}| = \|\widetilde{\overrightarrow{e_1}}\|^2$

Així com $\|\widetilde{\overrightarrow{e_1}} + \widetilde{\overrightarrow{e_2}}\|^2 = \|\widetilde{\overrightarrow{e_1}}\|^2 + \|\widetilde{\overrightarrow{e_2}}\|^2 + 2\|\widetilde{\overrightarrow{e_1}}\|.\|\widetilde{\overrightarrow{e_2}}\|$

Que expressat en coordenades canòniques o "de referència": $(\overrightarrow{e_1}, \overrightarrow{e_2})$:

$\|\widetilde{\overrightarrow{e_1}} + \widetilde{\overrightarrow{e_2}}\|^2 = ((\overrightarrow{e_1} + \overrightarrow{e_1}.Cos\theta)^2 + (0 + \overrightarrow{e_2}.Sin\theta)^2 =$

$= [(\overrightarrow{e_1})^2 + 2.(\overrightarrow{e_1})^2.Cos\theta + (\overrightarrow{e_1})^2.(Cos\theta)^2] +$

$+(0 + \overrightarrow{e_2}.Sin\theta)^2$ (②)

$\left\|\widetilde{\overrightarrow{e_1}}\right\|^2 + \left\|\widetilde{\overrightarrow{e_2}}\right\|^2$+2$\left\|\widetilde{\overrightarrow{e_1}}\right\|.\left\|\widetilde{\overrightarrow{e_2}}\right\|$=$(\overrightarrow{e_1}+0)^2 + [(\mathrm{Cos}\theta.\overrightarrow{e_1})^2 +$
$(\mathrm{Sin}\theta.\overrightarrow{e_2})^2]$+2.$\sqrt{(\overrightarrow{e_1})^2}.\sqrt{[(\mathrm{Cos}\theta.\overrightarrow{e_1})^2 + (\mathrm{Sin}\theta.\overrightarrow{e_2})^2}$ →

Com que són vectors unitaris, $(\overrightarrow{e_1})^2 = 1$, $(\overrightarrow{e_2})^2 = 1$

i $\overrightarrow{e_1}.\overrightarrow{e_2}$=0 i si $\vec{a} = (a_1, a_2)$ $\|\vec{a}\|$=$\sqrt{a_1^2 + a_2^2}$

→ 1+ 1.$(\mathrm{Cos}\theta)^2$+1. $(\mathrm{Sin}\theta)^2$+2.$\sqrt{1.(\mathrm{Cos}\theta)^2 + 1.(\mathrm{Sin}\theta)^2}$ (①)

Si ara igualem ambdós costats de la igualtat: (②)= (①)

1+2.Cosθ+ 1. $(\mathrm{Cos}\theta)^2$+1.$(\mathrm{Sin}\theta)^2$=1+1+2.1

$2(1+$ Cosθ)= 4

cosa que coincideix amb la <u>Desigualtat triangular:</u>

(1+Cosθ) ≤ 2 → $\left|\widetilde{\overrightarrow{e_1}}\widetilde{\overrightarrow{e_2}}\right| \leq \left\|\widetilde{\overrightarrow{e_1}}\right\|.\left\|\widetilde{\overrightarrow{e_2}}\right\|$

I ara: diferencial de <u>Volum</u>:

$dV^2 = \sqrt{g_{00}g_{11}g_{22}g_{33}}.dx^0dx^1dx^2dx^3$