EXPLICACIÓ DE L'EXPANSIÓ DE LA FUNCIÓ BETA:

(no és original meu)

B(x,y)=$\int_0^1 t^{x-1}(1-t)^{y-1}$dt

$(1-t)^{y-1}$=binomi de Newton=$\sum_{k=0}^{n}\binom{y-1}{k}.(1)^{y-1-k}(-1)^k(t)^k$

$\int_0^1 \sum_{k=0}^{n} t^{x-1}\binom{y-1}{k}.(1)^{y-1-k}(-1)^k(t)^k\,dt \rightarrow$

$\rightarrow$

$\int_0^1 t^{x-1}\binom{y-1}{0}.1^{y-1-0}(-1)^0t^0$.dt+

$+\int_0^1 t^{x-1}\binom{y-1}{1}.1^{y-1-1}(-1)^1t^1$.dt+

$+\int_0^1 t^{x-1}\binom{y-1}{2}.1^{y-1-2}(-1)^2t^2$.dt+

$+\int_0^1 t^{x-1}\binom{y-1}{3}.1^{y-1-3}(-1)^3t^3$.dt+...

...+ $\int_0^1 t^{x-1}\binom{y-1}{y-1}.1^0(-1)^{y-1}t^{y-1}$.dt

On $\binom{y-1}{0}=\frac{(y-1)!}{0!.(y-1-0)!}=1$

$\binom{y-1}{1}=\frac{(y-1)!}{1!.(y-1-1)!}=\frac{(y-1)!}{1!.(y-2)!}=\frac{(y-1)}{1!}$

$\binom{y-1}{2}=\frac{(y-1)!}{2!.(y-1-2)!}=\frac{(y-1).(y-2)}{2!}$

$\binom{y-1}{3}=\frac{(y-1)!}{3!.(y-1-3)!}=\frac{(y-1).(y-2).(y-3)}{3!}$

...

Per tant la integral és:

$$\frac{t^x}{x}.1+(-1).(y-1)\frac{t^{x+1}}{x+1}+(-1).\frac{(y-1)(y-2)}{2!}\frac{t^{x+2}}{x+2}$$
$$+(1).\frac{(y-1)(y-2)(y-3)}{3!}\frac{t^{x+3}}{x+3}|_0^1+\cdots$$

$$\ldots+\int_0^1(-1)^{y-1}.\frac{(y-1)!}{(y-1)!((y-1)-(y-1))!}.t^{x-1+(y-1)}.\text{dt}$$

Sabem que: B(x,y)=$\frac{\Gamma(x).\Gamma(y)}{\Gamma(x+y)}$

Podem operar la integral i otenir:

$$1.\frac{1}{x}-\frac{(y-1)}{1!.(x+1)}+\frac{(y-1)(y-2)}{2!.(x+2)}-\frac{(y-1)(y-2)(y-3)}{3!.(x+3)}+\cdots$$

$$\ldots+(-1)^{y-1}.1.\frac{1}{x+y-2}t^{x+y-2}|_0^1\rightarrow(-1)^{y-1}.1.\frac{1}{x+y-2}$$